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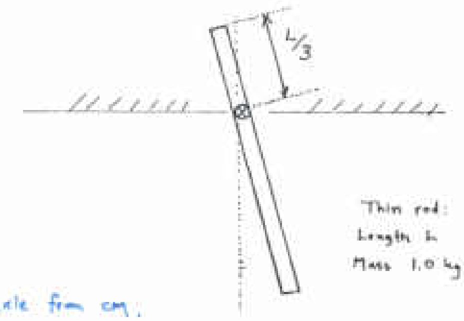
**Phys 221 (Section 8)**

**Quiz #5**




**1.** A uniform thin rod of length $L$ and mass 1.0 kg is mounted on a frictionless axle attached to the rod at a distance $\frac{1}{3}L$ from one of its ends and allowed to make small oscillations in a vertical plane, as diagrammed here.

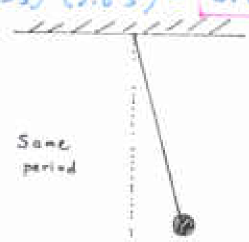
a) The period of oscillation is found to be 1.53 s. What is the length $L$ of the rod?

Use $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}}$ Here, $d = \left(\frac{L}{2} - \frac{L}{3}\right) = \frac{L}{6}$, dist. of axle from cm.

$I = I_{cm} + md^2 = \frac{1}{12}mL^2 + m\left(\frac{L}{6}\right)^2 = \left(\frac{1}{12} + \frac{1}{36}\right)mL^2 = \frac{1}{9}mL^2$

So $T = 2\pi\sqrt{\frac{\frac{1}{9}mL^2}{mg\frac{L}{6}}} = 2\pi\sqrt{\frac{2}{3}\frac{L}{g}}$ Solve for $L$: $T^2 = 4\pi^2\frac{2}{3}\frac{L}{g}$

$L = \frac{3}{8\pi^2}T^2 g = \frac{3}{8\pi^2}(1.53\,s)^2(9.8\,\frac{m}{s^2}) = \boxed{0.872\ m}$




b) If a simple pendulum is to have the same period, what must be the length of that simple pendulum?

For a simple pendulum,

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$ Again, $T = 1.53\,s$. Solve for $L$:

$T^2 = 4\pi^2\frac{L}{g}$ $L = \frac{T^2 g}{4\pi^2} = \frac{(1.53\,s)^2(9.8\,\frac{m}{s^2})}{4\pi^2} = 0.581\ m = \boxed{58.1\ cm}$

**2.** A wave function is given by:

$$y = (4.0\ \text{mm})\cos\left[(2.0\ \text{m}^{-1})x - (3.0\ \text{s}^{-1})t\right] .$$

a) Find the wavelength $\lambda$ and the speed $v$ of the wave.

We can read off: $A = 4.0$ mm $k = 2.0\ m^{-1}$ and $\omega = 3.0\ s^{-1}$ from this wavefunction. Then:

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{(2.0\,m^{-1})} = \boxed{3.14\ m}$ $v = \frac{\omega}{k} = \frac{3.0\ s^{-1}}{2.0\ m^{-1}} = \boxed{1.5\ \frac{m}{s}}$

b) Write down a suitable wavefunction for a harmonic wave which has all of the following properties (in comparison with the wave given above):

- Half the amplitude.
- The *same* wavelength but twice the wave speed.
- The *opposite* direction of wave motion.

We want a harmonic wave with $A = 2.0$ mm. Since $k = \frac{2\pi}{\lambda}$ and $\lambda$ is the same then $k$ is the same but since $v = \frac{\omega}{k}$, to get twice the wave speed we need to double $\omega$, so $\omega = 6.0\ s^{-1}$ here. Finally to change the direction of the wave, change the relative sign of the $kx$ and $\omega t$ terms. Thus, a suitable wave function is

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Formulae for pendula: $\omega = \sqrt{\frac{g}{L}}$ $\omega = \sqrt{\frac{mgd}{I}}$ $\omega = 2\pi f$

$g = 9.8\frac{\text{m}}{\text{s}^2}$ Mom. of In. of rod about CM: $I_{\text{CM}} = \frac{1}{12}ML^2$

Mom. of In. of rod of length $L$ about one end: $I = \frac{1}{3}ML^2$

$$\boxed{y = (2.0\ mm)\cos\left[(2.0\ m^{-1})x + (6.0\ s^{-1})t\right]}$$