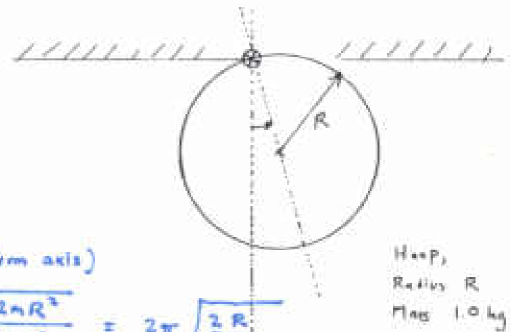
Name Earl E. Byrd

**Phys 221 (Section 6)**

**Quiz #5**




1. A hoop of mass 1.0 kg and radius $R$ is mounted on a frictionless axle (attached to the hoop itself) and allowed to make small oscillations in a vertical plane, as diagrammed here.

a) The period of oscillation is found to be 1.53 s. What is the radius of the hoop?

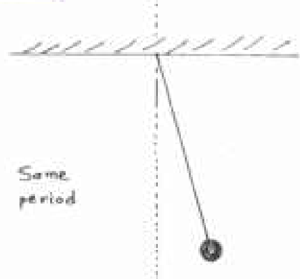
Use $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}}$. Here, $d = R$ (dist of cm from axis)

$I = I_{cm} + md^2 = mR^2 + mR^2 = 2mR^2$ So $T = 2\pi\sqrt{\frac{2mR^2}{mgR}} = 2\pi\sqrt{\frac{2R}{g}}$

Solve for R,

$T^2 = 4\pi^2\left(\frac{2R}{g}\right) \Rightarrow R = \frac{T^2 g}{8\pi^2} = \frac{(1.53\,s)^2(9.8\,\frac{m}{s^2})}{8\pi^2} = 0.291\text{ m} = \boxed{29.1\text{ cm}}$

b) If a simple pendulum is to have the same period, what must be the length of that simple pendulum?




For a simple pendulum,

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$ Again, $T = 1.53\,s$. Solve for L:

$T^2 = 4\pi^2\frac{L}{g}$ $L = \frac{T^2 g}{4\pi^2} = 0.581\text{m} = \boxed{58.1\text{ cm}}$

2. A wave function is given by:

$$y = (5.0\text{ mm})\sin\left[-(1.5\text{ m}^{-1})x + (4.0\text{ s}^{-1})t\right] \ .$$

a) Find the wavelength $\lambda$ and the speed $v$ of the wave.

We can read off: $A = 5.0$ mm, $k = 1.5\text{ m}^{-1}$ and $\omega = 4.0\text{ s}^{-1}$ from this wavefunction. Then:

$\lambda = \frac{2\pi}{k} = 2\pi/(1.5\text{ m}^{-1}) = \boxed{4.20\text{ m}}$

$v = \frac{\omega}{k} = \frac{4.0\text{ s}^{-1}}{1.5\text{ m}^{-1}} = \boxed{2.67\,\frac{m}{s}}$

b) Write down a suitable wavefunction for a harmonic wave which has all of the following properties (in comparison with the wave given above):

- Twice the amplitude
- The *same* wave speed but half the wavelength.
- The *opposite* direction of wave motion.

We want a harmonic wave with $A = 10.0$ mm. Since $k = \frac{2\pi}{\lambda}$, half the wavelength gives twice the value of k. But since $v = \frac{\omega}{k}$ and v remains the same we must also double $\omega$. So $k = 3.0\text{ m}^{-1}$, $\omega = 8.0\text{ s}^{-1}$. Finally to change the direction of wave motion, change the relative sign of the kx and ωt terms. Thus, a suitable wavefunction is:

$$\boxed{y = (10.0\text{ mm})\sin\left[(3.0\text{ m}^{-1})x + (8.0\text{ s}^{-1})t\right]}$$

---

Formulae for pendula: $\omega = \sqrt{\frac{g}{L}}$ $\omega = \sqrt{\frac{mgd}{I}}$ $\omega = 2\pi f$

$g = 9.8\frac{\text{m}}{\text{s}^2}$ Mom. of In. of hoop about CM: $I_{\text{CM}} = MR^2$