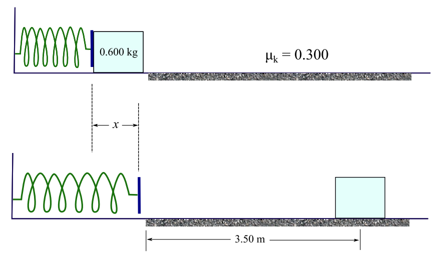
Name________________________________

Mar. 22, 2013

Quiz #3 — Spring 2013

Phys 2110 – Sec 4

1. A 0.600–kg mass is held against a spring of force constant $3500\,\frac{\mathrm{N}}{\mathrm{m}}$, compressing it by some amount x. The mass is released; after it leaves the spring it slides on a rough horizontal surface for which the coefficient of kinetic friction is 0.300. The mass slides 3.50 m before coming to rest.



a) What was the work done by friction on the mass?

While the mass slides on the surface, the normal force is $n = mg$ and the friction force has magnitude $\mu_k mg$. It moves a distance d *opposite* the direction of the force. The work done by friction is:

$$W_{\mathrm{fric}} = f_k d\cos(180^\circ) = (-1)\mu_k mgd = -(0.300)(0.600\ \mathrm{kg})(9.80\tfrac{\mathrm{m}}{\mathrm{s}^2})(3.50\ \mathrm{m}) = \boxed{-6.17\ \mathrm{J}}$$

b) By what length was the spring originally compressed?

Initially, all the energy was in the potential energy of the spring. In the final picture there is *no* mechanical energy. This gives

$$\Delta E = 0 - \tfrac{1}{2}kx^2 = W_{\mathrm{fric}} = -6.17\ \mathrm{J} \qquad \Longrightarrow \qquad x^2 = \frac{2(6.17\ \mathrm{J})}{k}$$

Plug in:

$$x^2 = \frac{2(6.17\ \mathrm{J})}{3500\,\frac{\mathrm{N}}{\mathrm{m}}} = 3.52\times 10^{-3}\ \mathrm{m}^2 \qquad \Longrightarrow \qquad x = 5.9\times 10^{-2}\ \mathrm{m} = \boxed{5.9\ \mathrm{cm}}$$

2. A particle moves in one dimension in a region where the potential energy function is given by

$$U(x) = 2.00x^2 - 14.0x + 15.0$$

where U is in J when x is in meters.

a) Find the point(s) of equilibrium for this potential energy function.

An equilibrium point is where $F_x = -\frac{dU}{dx} = 0$, so solve for x in

$$\frac{dU}{dx} = 0 = 4.00x - 14.0 \qquad \Longrightarrow \qquad x = \frac{14.0}{4.0}\ \mathrm{m} = \boxed{3.5\ \mathrm{m}}$$

b) Specify if the point(s) given in (a) are points of stable or unstable equilibrium

It is a point of **stable** equilibrium since it is clearly the *minimum* of the upward-facing parabola plot of $U(x)$.

c) If the total energy of the particle is 3.00 J, find the turning points of the motion.

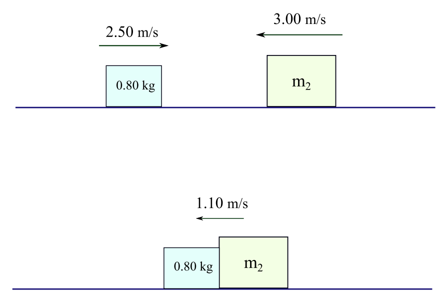
A turning point of the motion is a place where $K = 0$, which is where $E = U(x)$. Thus, solve for x in

$$E = 3.00\ \text{J} = U(x) = 2.00x^2 - 14.0x + 15.0 \qquad \Longrightarrow \qquad 2.00x^2 - 14.0x + 12.0 = 0$$

$$\Longrightarrow \qquad x^2 - 7.0x + 6.0 = 0 \qquad \Longrightarrow \qquad x = \frac{7.0 \pm \sqrt{49 - 24}}{2.0} \qquad \Longrightarrow \qquad x = \boxed{\begin{cases} 1.0\ \text{m} \\ 6.0\ \text{m} \end{cases}}$$

3. Two masses ($m_1 = 0.800$ kg and a second mass m_2) have a collision on a one-dimensional frictionless track. Before the collision m_1 moves to the right with speed $2.50\frac{\text{m}}{\text{s}}$ and m_2 moves to the left with speed $3.00\frac{\text{m}}{\text{s}}$.

After the collision, The masses are stuck together and move to the left with speed $1.10\frac{\text{m}}{\text{s}}$.



a) What is the value of m_2 ?

Momentum is consevered in the collision. This gives the condition

$$(0.800\ \text{kg})(2.50\tfrac{\text{m}}{\text{s}}) + m_2(-3.00\tfrac{\text{m}}{\text{s}}) = (0.800\ \text{kg} + m_2)(-1.10\tfrac{\text{m}}{\text{s}})$$

Solve for m_2:

$$[(0.800)(2.50) + (0.800)(1.10)]\ \tfrac{\text{kg}\cdot\text{m}}{\text{s}} = m_2[(3.00\tfrac{\text{m}}{\text{s}}) - (1.10\tfrac{\text{m}}{\text{s}})]$$

$$\Longrightarrow \qquad 2.88\ \tfrac{\text{kg}\cdot\text{m}}{\text{s}} = m_2(1.90\tfrac{\text{m}}{\text{s}}) \qquad \Longrightarrow \qquad m_2 = \boxed{1.52\ \text{kg}}$$

b) How much mechanical energy was lost (or gained!) in the collision?

Initial energy of the system is

$$K_1 = \tfrac{1}{2}(0.800\ \text{kg})(2.50\tfrac{\text{m}}{\text{s}})^2 + \tfrac{1}{2}(1.52\ \text{kg})(3.00\tfrac{\text{m}}{\text{s}})^2 = 9.34\ \text{J}$$

Final energy of the system is

$$K_2 = \tfrac{1}{2}(0.800\ \text{kg} + 1.52\ \text{kg})(1.10\tfrac{\text{m}}{\text{s}})^2 = 1.40\ \text{J}$$

The *loss* of energy was

$$-\Delta E = 9.34\ \text{J} - 1.40\ \text{J} = \boxed{7.9\ \text{J}}$$

You must show all your work and include the right units with your answers!

$$W = fd\cos\theta \qquad W = \int_a^b F_x\,dx \qquad K = \tfrac{1}{2}mv^2 \qquad U_{\text{grav}} = mgy \qquad U_{\text{spr}} = \tfrac{1}{2}kx^2$$

$$E = K + U \qquad \Delta E = \Delta K + \Delta U = W_{\text{non-cons}} \qquad F_x = -\frac{dU}{dx} \qquad f_k = \mu_k n$$

$$\mathbf{p} = m\mathbf{v} \qquad \mathbf{F}_{\text{ext, net}} = M\mathbf{a}_{\text{CM}} = \frac{d\mathbf{P}}{dt} \qquad \mathbf{J} = \Delta\mathbf{p} \qquad \overline{\mathbf{F}} = \frac{\Delta\mathbf{p}}{\Delta t} \qquad g = 9.80\tfrac{\text{m}}{\text{s}^2}$$