Name________________________________

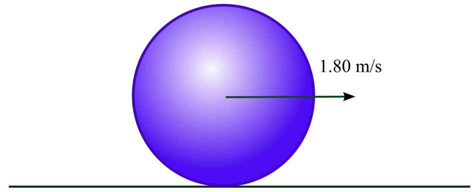
Apr. 18, 2012

Quiz #4 — Spring 2012

Phys 2110 – Sec 4

1. A *hollow* ball (i.e a spherical shell) of mass 0.800 kg and radius 7.00 cm rolls without slipping on a flat surface with a speed of $1.80\frac{\text{m}}{\text{s}}$.

What is its (total) kinetic energy?

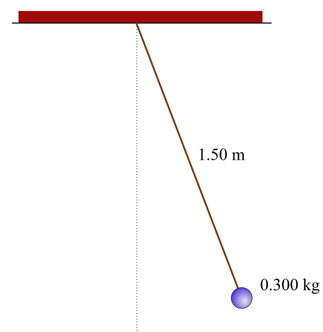


Use:

$$\begin{aligned} K &= K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}(\tfrac{2}{3}MR^2)\left(\frac{v}{R}\right)^2 \\ &= \left(\tfrac{1}{2}+\tfrac{1}{3}\right)Mv^2 = \tfrac{5}{6}(0.800\text{ kg})(1.80\tfrac{\text{m}}{\text{s}})^2 = \boxed{2.16\text{ J}} \end{aligned}$$

2. A simple pendulum has length 1.500 m with a small mass 300.0 g at the end. When it is makes small oscillations, we find that it makes 10 full swings in 25.23 s.

What is the value of g *as determined by this experiment*?



Use

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \Longrightarrow \qquad T^2 = 4\pi^2\frac{L}{g} \qquad \Longrightarrow \qquad g = \frac{4\pi^2 L}{T^2}$$

and since the measured period is

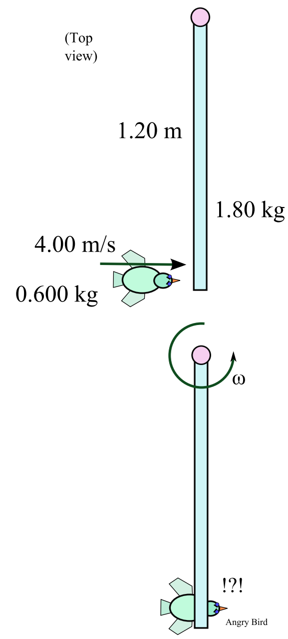
$$T = (25.23\text{ s})/10 = 2.523\text{ s}$$

we get

$$g = \frac{4\pi^2(1.50\text{ m})}{(2.523\text{ s})^2} = \boxed{9.30\tfrac{\text{m}}{\text{s}^2}}$$

3. A horizontal uniform rod of length 1.20 m and mass 1.80 kg swings freely about one end. In the figure to the right we are looking *down* on this rod; hint, gravity is not involved in this problem.

An angry bird of mass 0.600 kg flying at a speed of $4.00\frac{\text{m}}{\text{s}}$ hits the stationary rod at its far end and immediately gets stuck; then the rod/bird combination rotates about the axis with angular velocity ω.



a) Find the initial angular momentum of the bird.

We have solved the problem of a mass moving at speed along a line whose closest distance to the origin is b (given at end of quiz). Here, b is length of the stick, so result gives

$$L_{\text{bird}} = mvb = (0.600\text{ kg})(4.00\tfrac{\text{m}}{\text{s}})(1.20\text{ m}) = \boxed{2.88\,\tfrac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

b) Find the moment of inertia of rod/bird combination.

$$\begin{aligned} I &= I_{\text{stick}} + mL^2 = \tfrac{1}{3}ML^2 + mL^2 \\ &= \tfrac{1}{3}(1.80\text{ kg})(1.20\text{ m})^2 + (0.600\text{ kg})(1.20\text{ m})^2 = \boxed{1.73\,\text{kg}\cdot\text{m}^2} \end{aligned}$$

c) Find the final angular velocity ω of the system.

The system is isolated in regard to *torques* so that angular momentum is conserved. This gives

$$L_1 = L_2 \qquad \Longrightarrow \qquad L_{\text{bird}} = I\omega \qquad \Longrightarrow \qquad \omega = \frac{L_{\text{bird}}}{I}$$

Plug in the numbers,

$$\omega = \frac{(2.88\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}})}{(1.73\,\text{kg}\cdot\text{m}^2)} = \boxed{1.67\frac{\text{rad}}{\text{s}}}$$

You must show all your work and include the right units with your answers!

$$\tau = rF\sin\theta \qquad \tau_{\text{net}} = I\alpha \qquad I_{\text{cyl}} = \tfrac{1}{2}MR^2 \qquad I_{\text{sph, sol}} = \tfrac{2}{5}MR^2 \qquad I_{\text{sph, hol}} = \tfrac{2}{3}MR^2$$

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2 \qquad W_{\text{rot}} = \int \tau\, d\theta \qquad v_c = \omega r \qquad a_c = \alpha r \qquad K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = K_{\text{trans}} + K_{\text{rot}}$$

$$I_{\text{rod, end}} = \tfrac{1}{3}ML^2 \qquad L = I\omega \qquad L_{\text{str line}} = mvb \qquad \text{No net ext torque} \Rightarrow L \text{ conserved}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x \qquad \omega = \sqrt{\frac{k}{m}} \qquad T = \frac{2\pi}{\omega} \qquad f = \frac{1}{T} \qquad v_{\max} = A\omega \qquad a_{\max} = A\omega^2$$

$$\omega = \sqrt{\frac{g}{L}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \qquad \omega = \sqrt{\frac{mgL}{I}} \qquad T = 2\pi\sqrt{\frac{I}{mgL}} \qquad \omega = \sqrt{\frac{\kappa}{I}}$$