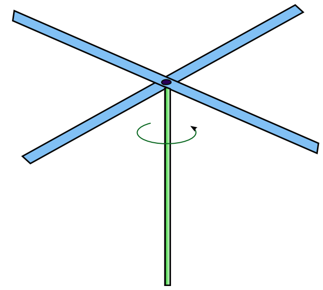
Name________________________________

Nov. 28, 2012

Quiz #4 — Fall 2012

Phys 2110 – Sec 5



1. A rotor is formed by taking two thin bars, each of length 2.00 m and mass 2.50 kg and rotating them about their centers.

Starting from rest, a constant torque is exerted on the rotor such that after 8.00 s, it has 855 J of kinetic energy.

a) What is final angular velocity of the rotor?

Find the moment of inertia of this contraption. It is that of two (identical) sticks rotating about their centers, thus

$$I = 2 \cdot \tfrac{1}{12} M L^2 = 2 \cdot \tfrac{1}{12}(2.50\ \mathrm{kg})(2.00\ \mathrm{m})^2 = 1.67\,\mathrm{kg \cdot m^2}$$

Then since the final KE is $855\ \mathrm{J}$,

$$K = \tfrac{1}{2} I \omega^2 = 855\ \mathrm{J} \qquad \Longrightarrow \qquad \omega^2 = \frac{2(855\ \mathrm{J})}{I} = \frac{2(855\ \mathrm{J})}{(1.67\,\mathrm{kg \cdot m^2})} = 1.03 \times 10^3\ \mathrm{s^{-2}}\ .$$

Then

$$\omega = \boxed{32.0 \tfrac{\mathrm{rad}}{\mathrm{s}}}$$

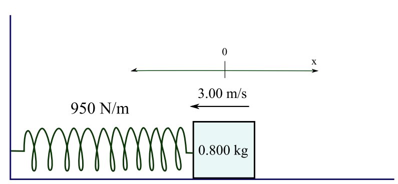
b) What was the torque that was exerted on the rotor?

The angular acceleration of the rotor was

$$\alpha = \frac{\Delta\omega}{t} = \frac{(32.0 \frac{\mathrm{rad}}{\mathrm{s}})}{8.00\ \mathrm{s}} = 4.00 \tfrac{\mathrm{rad}}{\mathrm{s^2}}$$

so the torque was

$$\tau = I\alpha = (1.67\,\mathrm{kg \cdot m^2})(4.00 \tfrac{\mathrm{rad}}{\mathrm{s^2}}) = \boxed{6.67\,\mathrm{N \cdot m}}$$



2. A 0.800–kg mass moves on a frictionless, flat surface and is attached to an ideal spring of force constant $950 \frac{\mathrm{N}}{\mathrm{m}}$. Initially it is at rest, but at $t = 0$ it is given a sudden kick such that it has a speed $3.00 \frac{\mathrm{m}}{\mathrm{s}}$ in the $-x$ direction.

a) Find the period of the resulting oscillatory motion.

Period of the mass-spring system is

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{0.800\ \mathrm{kg})}{(950\ \frac{\mathrm{N}}{\mathrm{m}})}} = \boxed{0.182\ \mathrm{s}}$$

b) Find the amplitude of the motion.

Conservation of energy gives (note, $3.00\,\frac{\mathrm{m}}{\mathrm{s}}$ is the maximum speed the mass has in the subsequent oscillations)

$$\tfrac{1}{2}mv_{\mathrm{max}}^2 = \tfrac{1}{2}kA^2 \qquad \Longrightarrow \qquad A^2 = \frac{mv_{\mathrm{max}}^2}{k} = \frac{(0.800\ \mathrm{kg})(3.00\frac{\mathrm{m}}{\mathrm{s}})^2}{(950\,\frac{\mathrm{N}}{\mathrm{m}})} = 7.58 \times 10^{-3}\ \mathrm{m}^2$$

Then

$$A = 8.71 \times 10^{-2}\ \mathrm{m} = \boxed{8.57\ \mathrm{cm}}$$

c) Write a function for its motion, $x(t)$.

When we released the mass from the maximum displacement, we could describe the motion with the cosine function, but that is not the case here. Here the mass starts at the $x = 0$ position but has negative velocity (the graph of $x(t)$ has negative slope) and this means we have a sine function with a $minus$ sign! Since the angular frequency is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{(0.182\ \mathrm{s})} = 34.5\ \mathrm{s}^{-1}$$

the equation which gives the motion must be

$$x(t) = -A\sin(\omega t) = \boxed{-(8.57\ \mathrm{cm})\sin((34.5\ \mathrm{s}^{-1})t)}$$

3. A harmonic wave moving in one-dimension is described by the function

$$y(x,t) = (0.015\ \mathrm{m})\cos\Big((177\ \mathrm{m}^{-1})\,x + (3.81\times 10^4\ \mathrm{s}^{-1})\,t\Big)$$

a) Find the wavelength and frequency of this wave.

Extract the quantities k and ω from the wave function. We have

$$k = 177\ \mathrm{m}^{-1} = \frac{2\pi}{\lambda} \qquad \Longrightarrow \qquad \lambda = \frac{2\pi}{k} = \frac{2\pi}{(177\ \mathrm{m}^{-1})} = 3.55\times 10^{-2}\ \mathrm{m} = \boxed{3.55\ \mathrm{cm}}$$

and

$$\omega = 3.81\times 10^4\ \mathrm{s}^{-1} = 2\pi f \qquad \Longrightarrow \qquad f = \frac{\omega}{2\pi} = \frac{(3.81\times 10^4\ \mathrm{s}^{-1})}{2\pi} = \boxed{6.06\times 10^3\ \mathrm{Hz}}$$

b) Find the speed of the wave.

$$v = \lambda f = (3.55\times 10^{-2}\ \mathrm{m})(6.06\times 10^3\ \mathrm{Hz}) = \boxed{215\tfrac{\mathrm{m}}{\mathrm{s}}}$$

c) Is the wave moving in the $+x$ or $-x$ direction? Explain how you decided this.

The wave is traveling in the $\boxed{-x \text{ direction}}$ (to the right); we know this because there is a $\boxed{\text{no sign difference}}$ between the x argument and the t argument of the cos function. (Consider the argument (phase) $x + vt$; if t increases, then x must decrease to maintain the same value. So the wave *form*, moves in the $-x$ direction with increasing time.)

You must show all your work and include the right units with your answers!

$$\tau = rF\sin\theta \qquad \tau_{\mathrm{net}} = I\alpha \qquad I_{\mathrm{cyl}} = \tfrac{1}{2}MR^2 \qquad I_{\mathrm{sph,\,sol}} = \tfrac{2}{5}MR^2 \qquad I_{\mathrm{sph,\,hol}} = \tfrac{2}{3}MR^2$$

$$K_{\mathrm{rot}} = \tfrac{1}{2}I\omega^2 \qquad W_{\mathrm{rot}} = \int \tau\, d\theta \qquad v_c = \omega r \qquad a_c = \alpha r \qquad K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = K_{\mathrm{trans}} + K_{\mathrm{rot}}$$

$$I_{\mathrm{rod,\,end}} = \tfrac{1}{3}ML^2 \qquad I_{\mathrm{rod,\,ctr}} = \tfrac{1}{12}ML^2 \qquad L = I\omega \qquad L_{\mathrm{str\ line}} = mvb \qquad \text{No net ext torque} \Rightarrow L \text{ cons'd}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x \qquad \omega = \sqrt{\frac{k}{m}} \qquad T = \frac{2\pi}{\omega} \qquad f = \frac{1}{T} \qquad v_{\max} = A\omega \qquad a_{\max} = A\omega^2$$

$$\omega = \sqrt{\frac{g}{L}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \qquad \omega = \sqrt{\frac{mgL}{I}} \qquad T = 2\pi\sqrt{\frac{I}{mgL}} \qquad \omega = \sqrt{\frac{\kappa}{I}}$$

$$\lambda f = v \qquad k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f \qquad y(x,t) = A\cos(kx \mp \omega t + \phi)$$