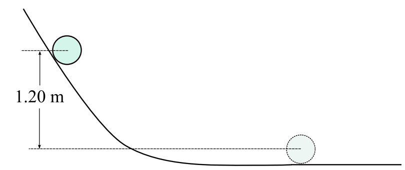
Name________________________

Nov. 30, 2011

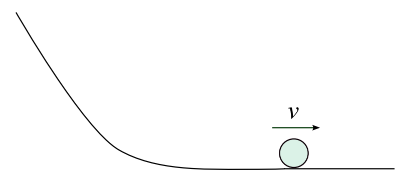
Quiz #4 — Fall 2011

Phys 2110 – Sec 4

1. A uniform cylinder of mass 0.600 kg and radius 5.00 cm begins at rest at a height of 1.20 m and rolls (smoothly and without slipping) onto a flat surface.



a) What is the *total* kinetic energy of the cylinder as it rolls on the flat surface?



By energy conservation it is just the potential energy which it had at the beginning, namely

$$K_{\rm roll} = U_{\rm init} = mgh = (0.600\ {\rm kg})((9.80\tfrac{\rm m}{\rm s^2})(1.2\ {\rm m}) = \boxed{7.06\ {\rm J}}$$

b) What is the speed of the cylinder (that is, its center) as it rolls on the flat surface?

The KE found in (a) has both translational and rotational parts , but they are related. We find:

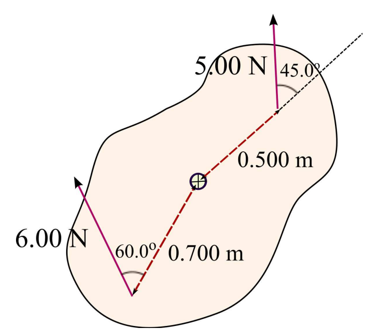
$$\begin{aligned} K_{\rm roll} &= K_{\rm tr} + K_{\rm rot} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 \\ &= \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 = \tfrac{3}{4}mv^2 \end{aligned}$$

Then

$$v = \sqrt{\frac{4}{3}\frac{K_{\rm roll}}{m}} = \sqrt{\frac{4}{3}\frac{(7.06\ {\rm J})}{(0.60\ {\rm kg})}} = \boxed{3.96\tfrac{\rm m}{\rm s}}$$

2. An oddly-shaped object rotates on a frictionless axle in the plane of the page; two forces are applied (in the plane of the page), with angles and distances from the axle as shown.

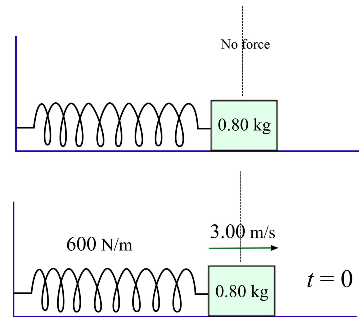
What is the net torque on the wheel? (be careful with signs and units. . .)



We note that the $5.00\ {\rm N}$ force gives a ccw (positive in our usage) torque while the $6.00\ {\rm N}$ force gives a negative torque. The total is

$$\tau_{\rm tot} = +(0.500\ {\rm m})(5.00\ {\rm N})\sin 45^\circ - (0.700\ {\rm m})(6.00\ {\rm N})\sin 60^\circ = \boxed{-1.87\,{\rm N\cdot m}}$$

3. A 0.800−kg mass is at rest and attached to the end of an ideal spring of constant $6.00 \times 10^2\ \frac{\text{N}}{\text{m}}$; it can move in one dimension on a frictionless surface. It is suddenly given an an impulse such that at $t = 0$ it has a velocity of $+3.00\frac{\text{m}}{\text{s}}$ (see fig).



a) What is the frequency of the resulting harmonic motion?

Irregardless, I say *irregardless* of the amplitude, the frequency of the motion of the mass-spring system will be

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{(600\ \frac{\text{N}}{\text{m}})}{0.800\ \text{kg}}} = \boxed{4.36\ \text{Hz}}$$

b) What is the amplitude of the resulting harmonic motion?

Conservation of energy between the starting point (maximum speed, no potential energy) and maximum extension of the spring gives

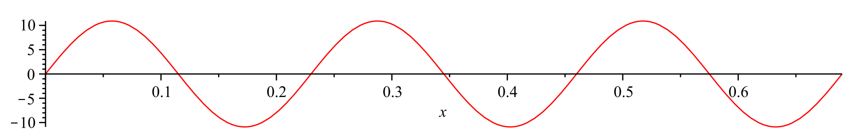
$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}kA^2 \qquad \Longrightarrow \qquad A = \sqrt{\frac{m}{k}}\,v_0$$

Plug in the numbers:

$$A = \sqrt{\frac{(0.800\ \text{kg})}{(600\ \frac{\text{N}}{\text{m}})}}\,(3.00\tfrac{\text{m}}{\text{s}}) = 0.109\ \text{m} = \boxed{10.9\ \text{cm}}$$

c) In the space below make a sketch of the function $x(t)$ (show several periods).

The graph will be sinusoidal but it must have $x(0) = 0$ and positive slope at $t = 0$. As the amplitude is $10.9\ \text{cm}$ and the period is $1/f = 0.23\ \text{s}$ it should look like:



d) Give an explicit (mathematical) expression for $x(t)$.

From part (c) it is clear that we want a sine function of the appropriate amplitude and angular frequency.

The angular frequency of the motion is $\omega = 2\pi f = 27.4\frac{\text{rad}}{\text{s}}$ and the amplitude is $10.9\ \text{cm}$ so the function we want is

$$x(t) = (10.9\ \text{cm})\sin([27.4\ \text{s}^{-1}]\,t)$$

You must show all your work and include the right units with your answers!

$$E = K + U \qquad K = \tfrac{1}{2}mv^2 \qquad U_{\rm spr} = \tfrac{1}{2}kx^2 \qquad U_{\rm grav} = mgy \qquad \tau = rF\sin\theta \qquad \tau = I\alpha$$

$$I_{\rm cyl} = \tfrac{1}{2}MR^2 \quad I_{\rm sph,\,sol} = \tfrac{2}{5}MR^2 \quad K = \tfrac{1}{2}I\omega^2 \quad v = \omega R \qquad K_{\rm roll} = K_{\rm tr} + K_{\rm rot} \quad \boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

$$L = I\omega \qquad L_{\rm str\ line} = mvb \qquad \frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x \qquad \omega = \sqrt{\frac{k}{m}} \qquad T = \frac{2\pi}{\omega} \qquad f = \frac{1}{T}$$

$$\omega = \sqrt{\frac{g}{L}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \qquad \omega = \sqrt{\frac{mgL}{I}} \qquad T = 2\pi\sqrt{\frac{I}{mgL}} \qquad \lambda f = v \qquad k = \frac{2\pi}{\lambda}$$