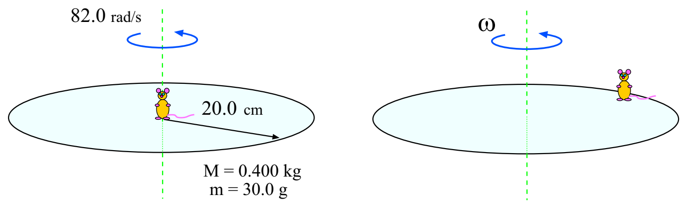
Name______________________________

Dec. 1, 2010

Quiz #4 — Fall 2010

Phys 2110 – Sec 3

1. a) A uniform disk of mass 0.400 kg and radius 20.0 cm rotates on a frictionless vertical axle at a rate of $82.0\frac{\text{rad}}{\text{s}}$ while a (small) 30.0 g mouse is sitting at the center of the disk.



The mouse decides to walk to the outer edge of disk. When he gets there, what is the new angular velocity of the disk?

Angular momentum is conserved; before and after it is a simple rotating system but the moment of inertia changes due to mouse motion. Initial and final moments of inertia of the system are

$$I_1 = I_{\text{disk}} + 0 = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(0.400\ \text{kg})(0.200\ \text{m})^2 = 8.00 \times 10^{-3}\,\text{kg}\cdot\text{m}^2$$

$$\begin{aligned} I_2 &= I_{\text{disk}} + I_{\text{mouse}} = \tfrac{1}{2}MR^2 + mR^2 = \tfrac{1}{2}(0.400\ \text{kg})(0.200\ \text{m})^2 + (0.030\ \text{kg})(0.200\ \text{m})^2 \\ &= 9.20 \times 10^{-3}\,\text{kg}\cdot\text{m}^2 \end{aligned}$$

Since $I_1\omega_1 = I_2\omega_2$, then

$$\omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{8.00 \times 10^{-3}}{9.20 \times 10^{-3}}(82.0\tfrac{\text{rad}}{\text{s}}) = 71.3\tfrac{\text{rad}}{\text{s}}$$

b) How much mechanical energy did the system lose or gain during the mouse's walk?

Initial and final (kinetic) energy is

$$E_1 = \tfrac{1}{2}I_1\omega_1^2 = \tfrac{1}{2}(8.00 \times 10^{-3}\,\text{kg}\cdot\text{m}^2)(82.0\ \text{s}^{-1})^2 = 26.9\ \text{J}$$

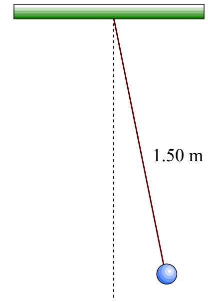
$$E_= \tfrac{1}{2}I_1\omega_2^2 = \tfrac{1}{2}(9.20 \times 10^{-3}\,\text{kg}\cdot\text{m}^2)(71.3\ \text{s}^{-1})^2 = 23.4\ \text{J}$$

Change in energy is

$$\Delta E = E_2 - E_1 = -3.5\ \text{J}$$

They system *lost* $\boxed{3.5\ \text{J}}$ of mechanical energy.

2. We have a simple pendulum of length 1.50 m and want to use it to determine the value of g. We let it undergo small oscillations and find that it makes 20 *complete* swings in 50.7 s.



What is the value of g that we determine *from this experiment*??

The period is $T = (50.7\ \mathrm{s})/50 = 2.535\ \mathrm{s}$. The period of a simple pendulum is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \qquad \Longrightarrow \qquad T^2 = 4\pi^2\frac{L}{g} \qquad \Longrightarrow \qquad g = \frac{4\pi^2 L}{T^2}$$

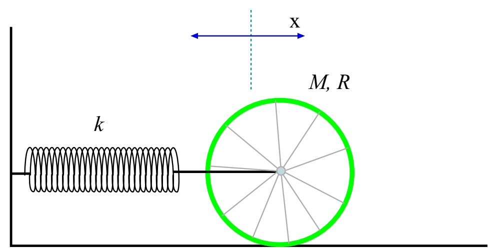
Plug in the numbers:

$$T = \frac{4\pi^2(1.5\ \mathrm{m})}{(2.535\ \mathrm{s})^2} = 9.21\frac{\mathrm{m}}{\mathrm{s}^2}$$

From this experiment we would be led to believe that g is $\boxed{9.21\frac{\mathrm{m}}{\mathrm{s}^2}}$.

3. One can show that a spring of force constant k attached to the axle of a hoop of mass M and radius R rolling on a flat surface gives a differential equation of the form

$$\frac{d^2x}{dt^2} = -\frac{k}{2M}x$$



Starting from this equation, write down an expression for the period T of the motion of this system. *Explain* your steps.

With the equation of motion in this form, ω^2 is given by the thing sitting in front of x (without the minus). Thus:

$$\omega^2 = \frac{k}{2M} \qquad \Longrightarrow \qquad \omega = \sqrt{\frac{k}{2M}} \qquad \Longrightarrow \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2M}{k}}$$

You must show all your work and include the right units with your answers!

$\tau = rF\sin\theta \quad \tau = I\alpha \quad K_{\mathrm{rot}} = \frac{1}{2}I\omega^2 \quad v_c = R\omega \quad a_c = R\alpha \quad K_{\mathrm{roll}} = \frac{1}{2}mv_c^2 + \frac{1}{2}I\omega^2 \quad I = \sum_i m_i r_i^2$

$I_{\mathrm{cyl}} = \frac{1}{2}MR^2 \quad I_{\mathrm{sph,sol}} = \frac{2}{5}MR^2 \quad I_{\mathrm{sph,hol}} = \frac{2}{3}MR^2 \quad I_{\mathrm{rod,\,ctr}} = \frac{1}{12}ML^2 \quad I_{\mathrm{rod,\,end}} = \frac{1}{3}ML^2$

$$L = I\omega \qquad \omega = 2\pi f \qquad f = \frac{1}{T} \qquad \omega = \sqrt{\frac{k}{m}} \qquad \omega = \sqrt{\frac{g}{L}} \qquad \omega = \sqrt{\frac{MgL}{I}}$$