Name______________________________

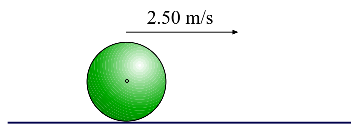
Nov. 21, 2008

Quiz #5 — Fall 2008

Phys 2110 – Sec 3

1. A hollow (uniform) sphere of mass 800. g rolls without slipping on a horizontal surface such that the speed of its center is $2.50\,\frac{\mathrm{m}}{\mathrm{s}}$.

What is the (total) kinetic energy of the sphere?



Kinetic energy is

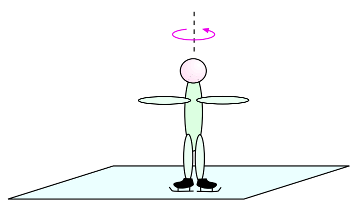
$$K_{\mathrm{roll}} = \tfrac{1}{2}mv_c^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv_c^2 + \tfrac{1}{2}\left(\tfrac{2}{3}mR^2\right)\left(\frac{v_c}{R}\right)^2 = \left(\tfrac{1}{2}+\tfrac{1}{3}\right)mv_c^2 = \tfrac{5}{6}mv_c^2$$

Plug in the numbers:

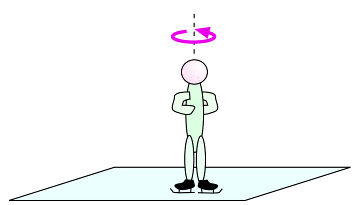
$$K_{\mathrm{roll}} = \tfrac{5}{6}(0.800\ \mathrm{kg})(2.50\,\tfrac{\mathrm{m}}{\mathrm{s}})^2 = 4.17\ \mathrm{J}$$

Total kinetic energy of the rolling sphere is $\boxed{4.17\ \mathrm{J}}$.

2. A skater's body has rotational inertia $4.00\,\mathrm{kg}\cdot\mathrm{m}^2$ with his hands held to his chest and $6.20\,\mathrm{kg}\cdot\mathrm{m}^2$ with his arms outstretched. The skater is twirling at $3.00\,\frac{\mathrm{rev}}{\mathrm{s}}$ with his arms held out. If he pulls his hands to his chest, how fast will he be twirling? (You can give the answer in $\frac{\mathrm{rev}}{\mathrm{s}}$.)

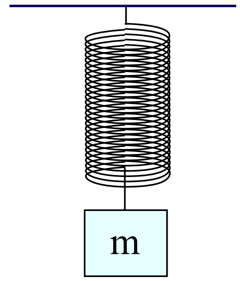


Angular momentum of the skater is conserved, that is, $I_i\omega_i = I_f\omega_f$. Here we can express ω in units of $\frac{\mathrm{rev}}{\mathrm{s}}$ (the conversion factor would cancel on both sides) and so



$$\omega_f = \frac{I_i\omega_i}{I_f} = \frac{(6.20\,\mathrm{kg}\cdot\mathrm{m}^2)(3.00\,\frac{\mathrm{rev}}{\mathrm{s}})}{(4.00\,\mathrm{kg}\cdot\mathrm{m}^2)} = 4.65\,\frac{\mathrm{rev}}{\mathrm{s}}$$

Final rotation rate of the skater is $\boxed{4.65\,\frac{\mathrm{rev}}{\mathrm{s}}}$.



3. A 200.0 g mass hung suspended from a particular (ideal, massless) spring makes small oscillations; we find that it makes 20 oscillations in 7.20 s.

a) What is the value of the spring constant?

The frequency is

$$f = (20 \text{ osc})/(7.20 \text{ s}) = 2.78 \text{ Hz}$$

Get the spring constant from

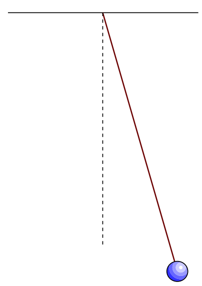
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \Longrightarrow \qquad k = 4\pi^2 f^2 m = 4\pi^2(2.78 \text{ s}^{-1})^2(0.200 \text{ kg}) = 60.9 \, \tfrac{\text{N}}{\text{m}}$$

b) What mass should we hang from this spring so that it makes oscillations with a frequency of 2.00 Hz?

Having k, use result from (a) to get new m for a frequency of 2.00 Hz:

$$k = 4\pi^2 f^2 m \qquad \Longrightarrow \qquad m = \frac{k}{4\pi^2 f^2} = \frac{(60.9 \, \frac{\text{N}}{\text{m}})}{4\pi^2(2.00 \text{ s}^{-1})^2} = 0.386 \text{ kg} = 386 \text{ g}$$

4. What is the length of a simple pendulum which has a period of 3.00 s?



Find the length from

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \Longrightarrow \qquad L = \frac{T^2 g}{4\pi^2} = \frac{(3.0 \text{ s})^2(9.80 \, \frac{\text{m}}{\text{s}^2})}{4\pi^2} = 2.23 \text{ m}$$

You must show all your work and include the right units with your answers!

$$\tau = rF\sin\theta \qquad \tau = I\alpha \qquad I_{\text{cyl}} = \tfrac{1}{2}MR^2 \qquad I_{\text{sph, solid}} = \tfrac{2}{5}MR^2 \qquad I_{\text{sph, hol}} = \tfrac{2}{3}MR^2$$

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2 \qquad v_c = \omega R \qquad a_c = \alpha R \qquad K_{\text{roll}} = K_{\text{trans}} + K_{\text{roll}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

$$\vec{\tau} = \vec{r}\times\vec{F} \qquad \vec{L} = \vec{r}\times\vec{p} \qquad L = I\omega \qquad \frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x \qquad \omega = \sqrt{\frac{k}{m}} \qquad T = \frac{2\pi}{\omega} \qquad f = \frac{1}{T}$$

$$\omega = \sqrt{\frac{g}{L}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \qquad \omega = \sqrt{\frac{mgL}{I}} \qquad T = 2\pi\sqrt{\frac{I}{mgL}} \qquad \omega = \sqrt{\frac{\kappa}{I}} \qquad g = 9.80 \, \tfrac{\text{m}}{\text{s}^2}$$