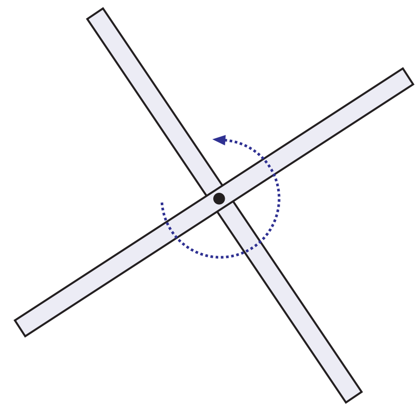
Name______________________________

Apr. 19, 2006

Quiz #5 — Spring 2006
Phys 2110

1. Two thin uniform rods, each of length 0.80 m and mass 2.00 kg are stuck together at their midpoints; we rotate this object about an axis though the midpoints and perpendicular to the plane of the object.

The object starts from rest and is acted on by a constant torque. In 8.0 s it has a rotation rate of 150 revolutions per minute.



a) What is the angular acceleration of the object?

Here we have

$$\omega_f = \left(150\tfrac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 15.7\tfrac{\text{rad}}{\text{s}}$$

and the initial angular velocity was zero, so the angular acceleration is

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{(15.7\frac{\text{rad}}{\text{s}} - 0\frac{\text{rad}}{\text{s}})}{8.0\text{ s}} = 1.96\tfrac{\text{rad}}{\text{s}^2}$$

b) What is the moment of inertia of the object?

It is sum of the moments of inertia of *two* such sticks about the center of mass, hence

$$I = 2\cdot\tfrac{1}{12}ML^2 = \tfrac{1}{6}(2.00\text{ kg})(0.80\text{ m})^2 = 0.213\,\text{kg}\cdot\text{m}^2$$

c) What was the magnitude of the torque applied to the object?

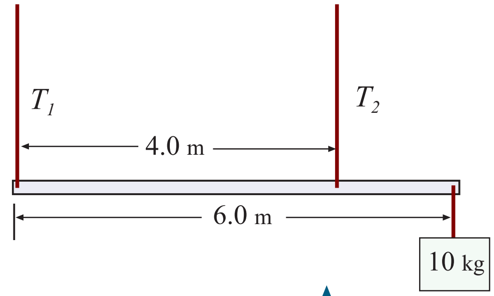
$$\tau = I\alpha = (0.213\,\text{kg}\cdot\text{m}^2)(1.96\tfrac{\text{rad}}{\text{s}^2}) = 0.419\,\text{N}\cdot\text{m}$$

d) At $t = 8.0$ s, what was the kinetic energy of the object?

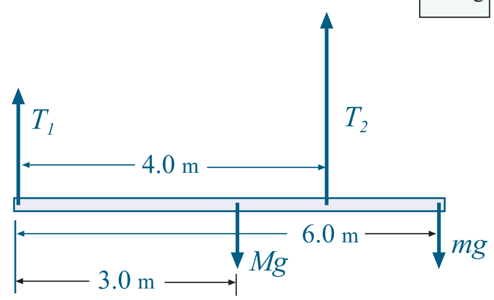
$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(0.213\,\text{kg}\cdot\text{m}^2)(15.7\tfrac{\text{rad}}{\text{s}})^2 = 26.3\text{ J}$$

2. A thin uniform rod of length 6.0 m and mass 80.0 kg is horizontal and is supported by two vertical ropes, one attached at the left end of the rod and the other attached 4.0 m from the left end. In addition, there is a 10.0 kg mass hanging from the right end.

Find the tensions in the two ropes.



Forces on the rod are as shown at the right. there are upward forces from the two rope tensions; the weight of the rod Mg is applied at its center and the weight of the hanging mass mg is applied at the end.



The total force on the rod is zero, so

$$T_1+T_2-Mg-mg=0 \quad \Longrightarrow \quad T_1+T_2=(M+m)g=882\ \text{N}$$

Choose the left end of the rod as the axis; the sum of torques about this point is zero. This gives:

$$-Mg(3.0\ \text{m})+T_2(4.0\ \text{m})-mg(6.0\ \text{m})=0 \qquad \text{or:}$$

$$T_2(4.0\ \text{m})=Mg(3.0\ \text{m})+mg(6.0\ \text{m})=2940\,\text{N}\cdot\text{m}$$

And this gives

$$T_2=735\ \text{N}$$

Putting this into the first result gives

$$T_1=882\ \text{N}-735\ \text{N}=147\ \text{N}$$

So $\boxed{T_1=147\ \text{N}}$ and $\boxed{T_2=735\ \text{N}}$.

You must show all your work and include the right units with your answers!

$$\omega_f=\omega_i+\alpha\Delta t \qquad \theta_f=\theta_i+\omega_i\Delta t+\tfrac{1}{2}\alpha(\Delta t)^2 \qquad \omega_f^2=\omega_i^2+2\alpha\Delta\theta$$

$$v_t=r\omega \qquad a_t=r\alpha \qquad \tau=|rF\sin\theta| \qquad \tau=I\alpha \qquad K=\tfrac{1}{2}I\omega^2 \qquad I=\sum_i m_i r_i^2$$

$$I_{\text{rod, ctr}}=\tfrac{1}{12}ML^2 \qquad I_{\text{rod, end}}=\tfrac{1}{3}ML^2 \qquad I_{\text{sph, sol}}=\tfrac{2}{5}MR^2 \qquad I_{\text{sph, sh}}=\tfrac{2}{3}MR^2$$

$$\text{Statics:} \quad \sum\mathbf{F}=0 \qquad \sum\tau=0$$