PHYSICS 2110 – EXAM #2
April 2, 2013

SEAT NO.______________ **NAME (PRINT)**____________________________

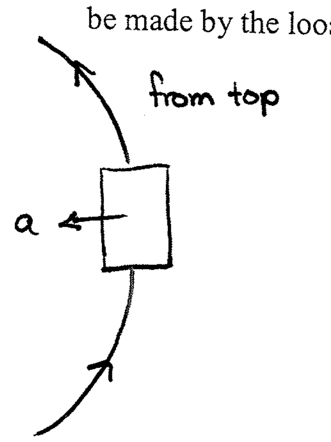
YOU MUST SHOW YOUR WORK AND EXPLAIN YOUR REASONING TO RECEIVE CREDIT. ALL CELL PHONES AND OTHER COMMUNICATION DEVICES MUST BE TURNED OFF AND STORED OUT OF SIGHT. NO EXTRA PAPERS ARE ALLOWED OTHER THAN THE PROVIDED FORMULA SHEET. STANDARD SCIENTIFIC CALCULATORS MAY BE USED.

You may ignore air resistance unless told otherwise.
Free-body diagrams are ***required*** for problems involving forces.

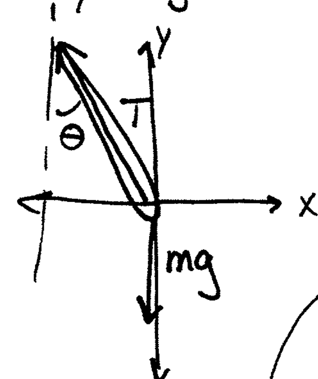
INSTRUCTORS (Circle ONE):	CLASS MEETING TIME
Shriner	8:00 AM
Kozub	9:05 AM
Kidd	10:10 AM
Murdock	11:15 AM
Ayik	1:25 PM

PROBLEM	POINT VALUE	YOUR SCORE
1	5	
2	15	
3	7	
4	4	
5	11	
6	5	
7	4	
8	9	
9	10	
10	10	
11	20	
TOTAL	100	

1. An old streetcar rounds an unbanked curve of radius 9.1 m, at 4.4 m/s. What angle with the vertical will be made by the loosely hanging hand straps? (5 pts)

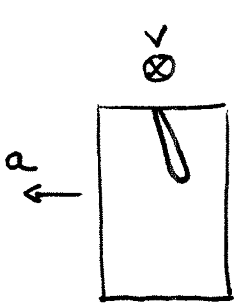


Free body diagram:

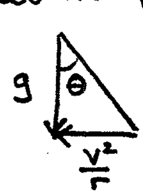


$$\Sigma F_y = T_y - mg = \cancel{m}a_y^{\,0}$$
$$T_y = mg$$
$$T\cos\theta = mg$$
$$T = \frac{mg}{\cos\theta}$$
$$\Sigma F_x = -T_x = -\frac{mv^2}{r}$$
$$-T\sin\theta = -\frac{mv^2}{r}$$
$$+\frac{\cancel{m}g}{\cos\theta}\sin\theta = +\frac{\cancel{m}v^2}{r}$$
$$\tan\theta = \frac{v^2}{gr}$$
$$\theta = \arctan\left(\frac{(4.4\,\frac{m}{s})^2}{(9.8\frac{m}{s^2})(9.1\,m)}\right)$$
$$\theta = 12.2^\circ \approx 12^\circ \text{ (2 s.f.)}$$

from back

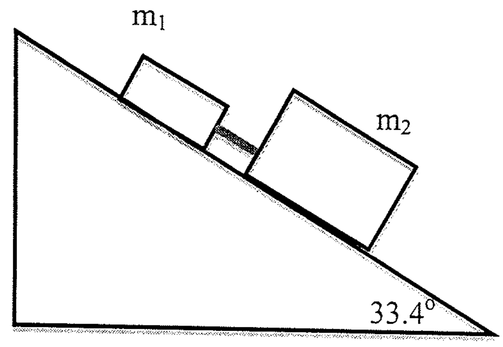


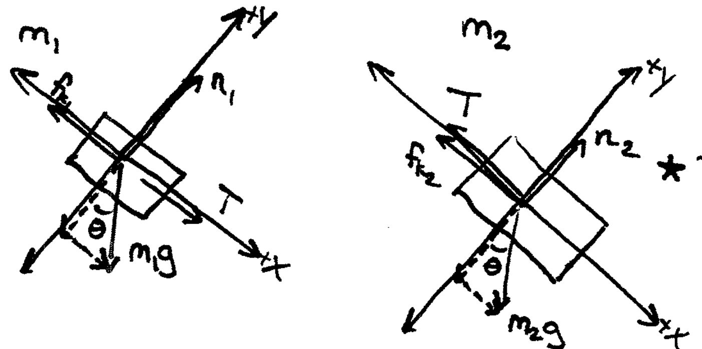
(also accepted:



$$\tan\theta = \frac{v^2/r}{g}$$
$$\theta = 12.2^\circ \approx 12^\circ$$)

2. A box of mass $m_1 = 1.65$ kg and a box of mass $m_2 = 3.30$ kg slide down an inclined plane while attached by a massless rod parallel to the plane. The angle of incline is 33.4°. The coefficient of friction between box 1 and the plane is $\mu_1 = 0.226$ and the coefficient of friction between box 2 and the plane is $\mu_2 = 0.113$. Find the tension in the rod and the acceleration of the two boxes. (15 points)





★ Draw a free-body diagram!! ★

$\Sigma F_{1y} = n_1 - m_1 g\cos\theta = \cancel{m_1 a_y}^{0}$

$n_1 = m_1 g\cos\theta$

$\Sigma F_{2y} = n_2 - m_2 g\cos\theta = \cancel{m_2 a_y}^{0}$

$n_2 = m_2 g\cos\theta$

$\Sigma F_{1x} = T + m_1 g\sin\theta - f_{k_1} = m_1 a_x$

$\Sigma F_{2x} = m_2 g\sin\theta - T - f_{k_2} = m_2 a_x$

a_x same for both boxes b/c joined by rod. T same for both.

Solve ΣF_{1x} for T

$T = m_1 a_x - m_1 g\sin\theta + f_{k_1}$

Sub into ΣF_{2x}

$m_2 g\sin\theta - (m_1 a_x - m_1 g\sin\theta + f_{k_1}) - f_{k_2} = m_2 a_x$

Solve for a_x

$(m_1 + m_2)a_x = m_2 g\sin\theta + m_1 g\sin\theta - f_{k_1} - f_{k_2}$

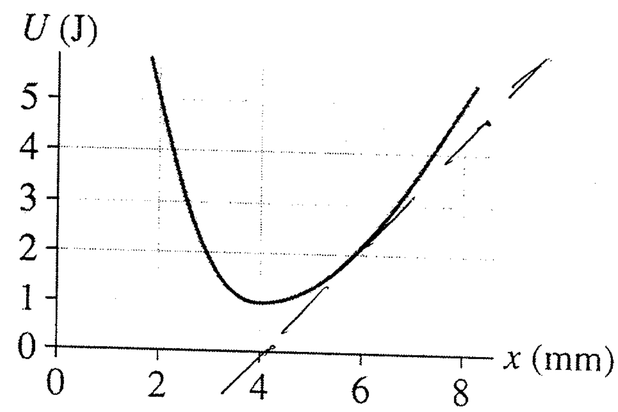
$a_x = \dfrac{(m_1+m_2)g\sin\theta - \mu_1 m_1 g\cos\theta - \mu_2 m_2 g\cos\theta}{m_1+m_2}$

$\boxed{a_x = 4.16\ m/s^2}$

Sub a_x into either ΣF_{1x} or ΣF_{2x} to get T

$\boxed{T = 1.01\ N}$

3. A particle with mass 2.0 g is subject to a single conservative force whose potential energy function is shown in the figure to the right. The particle oscillates along the x-axis between x = 2.0 mm and x = 8.0 mm.



(a) Find the maximum speed of the particle. (4 pts)

Turning points at x = 2.0 mm and x = 8.0 mm

$\Rightarrow E_{mech} = 5.0\,J$, $U_{min} = 1.0\,J \Rightarrow K_{max} = 4.0\,J$

$\Rightarrow 4.0\,J = \frac{1}{2}(.002\,kg)\,v_{max}^2 \Rightarrow v_{max} = 63\ m/s$

(b) Estimate the force on the particle at x = 6.0 mm. (3 pts)

$F_x = -\dfrac{dU}{dx}$. See tangent line: $\dfrac{dU}{dx} \approx \dfrac{4.2\,J - 0.0\,J}{.0080\,m - .0040\,m} = 1.0\times10^3\,N$

$\Rightarrow \vec{F} = -1.0\times10^3\,N\,\hat{i}$

4. Two vectors are given by $\vec{A} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\vec{B} = 3\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$. Calculate the angle between these two vectors. (4 pts)

$$\cos\alpha = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{6-12-20}{\sqrt{2^2+3^2+4^2}\sqrt{3^2+4^2+5^2}} = \frac{-26}{\sqrt{29}\sqrt{50}}$$

$$\alpha = 133^\circ$$

5. A force is given by $\vec{F} = [(3.00x - 4.00y)\hat{\imath} + 4.00x^2\hat{\jmath}]$. The force units are Newtons when x and y are in meters.

(a) Calculate the work done by this force when acting on a 1.50 kg mass that moves along the x-axis from $x = -1.00$ m to $x = +2.00$ m. (5 pts)

$$W = \int_{x=-1m}^{2m} \vec{F}\cdot d\vec{r} = \int_{-1m}^{2m} [(3x-4y)\hat{i} + 4x^2\hat{j}]\cdot\hat{i}\,dx = \frac{3}{2}x^2 - 4xy\Big|_{-1m}^{2m}$$

($y = 0$ along x-axis)

$$\therefore W = \frac{3}{2}(4-1) - 4(0)\ J = \frac{9}{2}\ J = 4.50\ J$$

(b) The speed of the mass at x=-1.00 m is 5.00 m/s and its speed at x=+2.00 m is 3.00 m/s. Do the appropriate calculations to determine whether the force in part (a) is the only force doing work on this mass. Describe the effect of any other forces that may be present. (6 pts)

$$\Delta K = W_{net} = K_f - K_i = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}(1.5\,kg)\left[\left(3\tfrac{m}{s}\right)^2 - \left(5\tfrac{m}{s}\right)^2\right]$$

$W_{net} = -12.0\ J < 4.50\ J$, so other forces are doing $-12.0\ J - 4.50\ J = -16.5\ J$ of work.

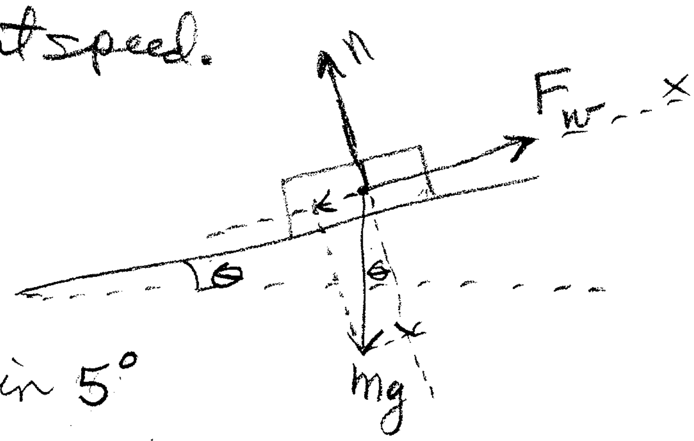
6. A 2000.-kg car can climb a 5.00° incline at a maximum speed of 35.0 m/s. Calculate the maximum power output to the wheels of the car. (5 pts)

$\Sigma F_x = F_w - mg\sin\theta = 0$ at constant speed.

$F_w = mg\sin\theta$

$P_{max} = F_w v = mgv\sin\theta$

$= (2000\,kg)(9.8\,m/s^2)(35\,m/s)\sin 5^\circ$



$P = 59{,}800\ W = 59.8\ kW\ (= 80.1\ hp)$

7. Indicate True or False for each of the following statements by writing T or F to the left of each statement: (1 pt each)

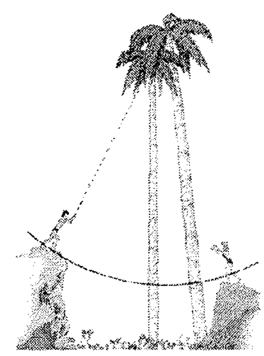
F (a) Only conservative forces can do work.

F (b) If only conservative forces act on a particle, the kinetic energy of the particle remains constant.

F (c) The work done by a conservative force equals the change in potential energy associated with that force.

F (d) A particle moves along the x-axis. If the potential energy associated with a conservative force decreases as the particle moves to the right, then the force points to the left.

8. Tarzan believes that Jane needs rescuing and (surprise!) grabs a vine to swing across the space between two rocks. The vine is 18.0 m long, and the lowest point of Tarzan's trajectory is 3.2 meters lower than his starting point on the rock. If the vine will break under a force of more than 920 N, what is the maximum mass that Tarzan can have so that the vine will not break before he reaches Jane? (9 pts)



$\uparrow T$

$\downarrow mg$

$$T - mg = \frac{m v_{bottom}^2}{r}$$

$$\Rightarrow T = mg + \frac{m v_{bottom}^2}{r} \Rightarrow m_{max} = \frac{T_{max}}{g + \frac{v_{bottom}^2}{r}}$$

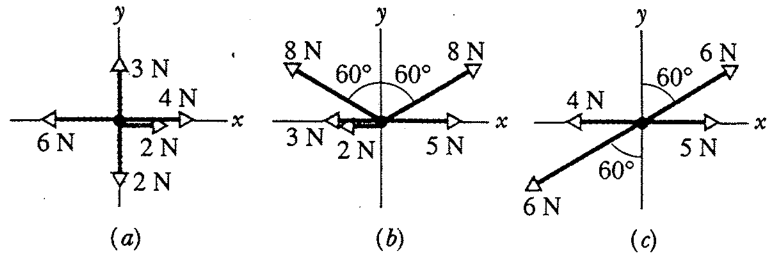
Only force doing work as Tarzan swings is gravity $\Rightarrow$

$$E_{mech} \text{ conserved} \Rightarrow mg(3.2\,m) + \tfrac{1}{2}m(0\,m/s)^2 = mg(0\,m) + \tfrac{1}{2} m v_{bottom}^2$$

$$\Rightarrow v_{bottom} = \sqrt{2g(3.2\,m)} = 7.9\,m/s$$

$$m_{max} = \frac{920\,N}{9.80\,m/s^2 + \frac{(7.9\,m/s)^2}{18.0\,m}} = 69\,kg$$

(a) The free-body diagrams in the figure to the right give, from overhead views, the horizontal forces acting on three boxes of chocolates as the boxes moves over a frictionless surface. For each box, state which components of linear momentum are conserved? (3 pts)

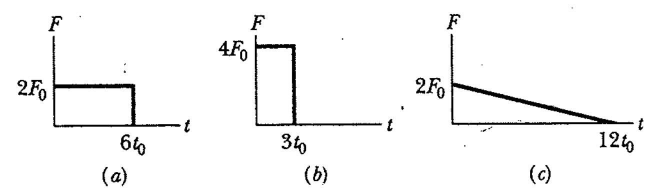


(a) x- component

(b) x- component

(c) y- component

(b) The figure to the right shows graphs of force magnitude versus time for a body involved in a collision. Rank the graphs according to the magnitude of the impulse on the body, greatest first. (3 pts)



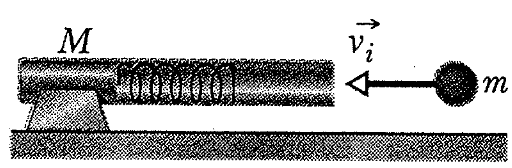
all the same

(c) A 0.70 kg ball moving horizontally at 5.0 m/s strikes a vertical wall. It rebounds immediately after the strike with a horizontal speed 2.0 m/s. What is the magnitude of the change in its linear momentum? (4 pts)

$$\Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = 0.7(-2.0-5.0)\,\vec{i}$$

$$\boxed{\Delta p = 4.9\ \tfrac{m}{s}\cdot kg}$$

10. A ball of mass $m = 0.5$ kg is shot with a speed of $v_i = 20$ m/s into the barrel of a spring gun, as shown to the right. The spring gun has a mass $M = 3.5$ kg and it is initially at rest on a frictionless surface. The ball sticks in the barrel at the point of maximum compression of the spring. Neglect the friction between the ball and the barrel while the ball is in motion.



(a) Determine the velocity of the center of mass before the ball collides with the spring gun. (3 pts)

$$V_{cm} = \frac{m v_i + M\times 0}{m+M} = \frac{0.5\times 20}{0.5+3.5} = \boxed{2.5\ m/s}$$

(b) What is the speed of the spring gun after the ball stops in the barrel? (2 pts)

$$\boxed{V_f = 2.5\ m/s}$$

(c) How much elastic energy is stored in the spring during the collision? (5 pts)

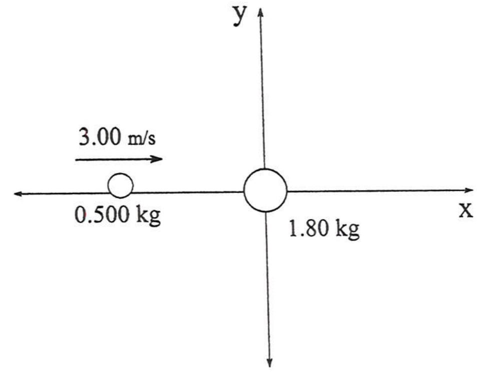
$$\tfrac{1}{2} m v_i^2 = \tfrac{1}{2}(m+M)\,V_f^2 + U$$

$$U = \tfrac{1}{2}\,0.5\times(20)^2 - \tfrac{1}{2}(0.5+3.5)(2.5)^2$$

$$\boxed{U = 87.5\ J}$$

11. Two hockey pucks (of masses 0.500 kg and 1.80 kg) are free to move on a horizontal frictionless table; they undergo a collision as seen from above in the pictures given here.

Before the collision the 0.500 kg mass moves in the +x direction with a speed of 3.00 m/s and the 1.80 kg mass is at rest. After the collision, the smaller mass moves in a direction 120.0° (ccw) from the +x axis with a speed of 1.80 m/s.



(a) Find the velocity components of the larger mass after the collision. (9 pts)

Cons of P_x:

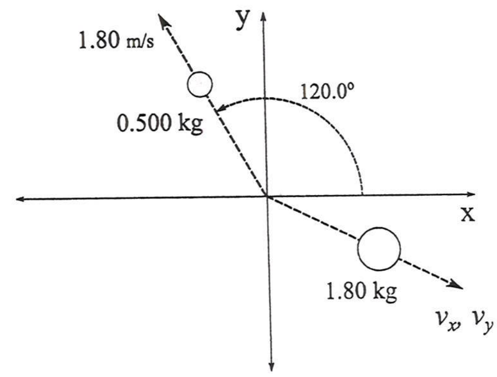
$$(0.50\,kg)(3.00\tfrac{m}{s}) = (0.50\,kg)(1.80\tfrac{m}{s})\cos(120°) + (1.80\,kg)V_x$$

$$\rightarrow \boxed{V_x = 1.08\,\tfrac{m}{s}}$$

Cons of P_y:

$$0 = (0.50\,kg)(1.80\tfrac{m}{s})\sin(120°) + (1.80\,kg)V_y$$

$$\rightarrow \boxed{V_y = -0.433\,\tfrac{m}{s}}$$



(b) Find the speed and direction of motion of the larger mass after the collision. (4 pts)

$$V = \sqrt{V_x^2 + V_y^2} = \boxed{1.16\,\tfrac{m}{s}}$$

$$\tan\theta = V_y / V_x = -0.401 \Rightarrow \theta = \boxed{-21.8°}$$

(c) How much energy was lost (or gained!) in the collision? (4 pts)

Before coll:

$$E_i = \tfrac{1}{2}(0.50\,kg)(3.0\tfrac{m}{s})^2 = 2.25\,J$$

After coll:

$$E_f = \tfrac{1}{2}(0.50\,kg)(1.80\tfrac{m}{s})^2 + \tfrac{1}{2}(1.80\,kg)(1.16\tfrac{m}{s})^2 = 2.02\,J$$

This is a $\boxed{\text{loss of } 0.23\,J}$

(d) What was the velocity of the center of mass before the collision? What was it after the collision? (3 pts)

$$\vec{V}_{cm} = \vec{P}/M \underset{\text{Before collision}}{=} \frac{(0.50\,kg)(3.00\tfrac{m}{s})}{(2.3\,kg)}\,\hat{i} = \boxed{(0.65\,\tfrac{m}{s})\,\hat{i}}$$

Was the $\boxed{\text{same value after}}$ the collision because $\vec{P}$ stays the same.