PHYSICS 2110 – EXAM #2
November 8, 2012

SEAT NO.____________ **NAME (PRINT)** Key

YOU MUST SHOW YOUR WORK AND EXPLAIN YOUR REASONING TO RECEIVE CREDIT. ALL CELL PHONES AND OTHER COMMUNICATION DEVICES MUST BE TURNED OFF AND STORED OUT OF SIGHT. NO EXTRA PAPERS ARE ALLOWED OTHER THAN THE PROVIDED FORMULA SHEET. STANDARD SCIENTIFIC CALCULATORS MAY BE USED.

You may ignore air resistance unless told otherwise.
Free-body diagrams are ***required*** for problems involving forces.

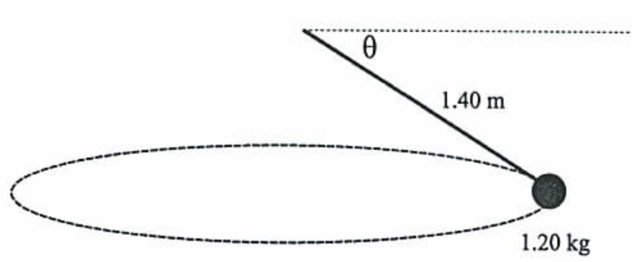
INSTRUCTORS (Circle ONE):	CLASS MEETING TIME
Shriner	8:00 AM
Kozub	9:05 AM
Kidd	10:10 AM
Kidd	11:15 AM
Murdock	12:20 PM

PROBLEM	POINT VALUE	YOUR SCORE
1	3	
2	11	
3	9	
4	20	
5	11	
6	9	
7	10	
8	10	
9	13	
10	4	
TOTAL	100	

1. State the physical condition that must be met for the momentum of a system to be conserved. (3 pts)

The momentum of a system is conserved if the net external force on the system is zero.

2. A 1.20 kg rock is whirled in a horizontal circle at the end of a 1.40-m long string. The tension in the string is 95.0 N



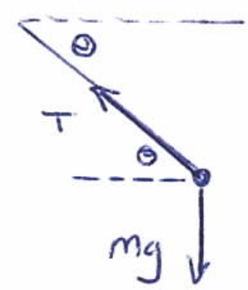
(a) What angle does the string make with the horizontal? (5 pts)

Force diagram is shown!
Vertical components sum to zero, so

$T\sin\Theta - mg = 0 \rightarrow \sin\Theta = \frac{mg}{T}$

$\rightarrow \sin\Theta = \frac{(1.20\,kg)(9.80\,\frac{m}{s^2})}{(95.0\,N)} = 0.124$

$\rightarrow \boxed{\Theta = 7.11^\circ}$



(b) What is the speed of the rock? (4 pts)

The horizontal (inward) component of the force is the "centripetal force": $T\cos\Theta = \frac{mv^2}{r}$ w/ $r = (1.40\,m)\cos\Theta = 1.39\,m$

$v^2 = \frac{rT\cos\Theta}{m} = \frac{(1.39\,m)(95.0\,N)\cos 7.11^\circ}{(1.20\,kg)} = 1.09\times10^2\,\frac{m^2}{s^2}$

$v = \boxed{10.4\,\frac{m}{s}}$

(c) How long does it take the rock to move around the circle (the period of the motion)? (2 pts)

$V = \frac{2\pi r}{T}$ (= Dist around circ/Period)

$T = \frac{2\pi r}{v} = \frac{2\pi(1.39\,m)}{10.4\,\frac{m}{s}} = \boxed{0.840\,s}$

3. A merry-go-round starts from rest and accelerates uniformly such that in 31.0 sec it has made 3.00 complete revolutions.

(a) What is its final angular velocity? (3 pts)

$3.00\,rev = (3.00\,rev)\left(\frac{2\pi\,rad}{1\,rev}\right) = 18.8\,rad$

As α is constant we can use

$\Theta - \Theta_0 = \frac{1}{2}(\omega_0 + \omega)t = \frac{\omega t}{2} \rightarrow \omega = \frac{2\Theta}{t} = \frac{2(18.8\,rad)}{31.0\,s} = \boxed{1.22\,\frac{rad}{s}}$

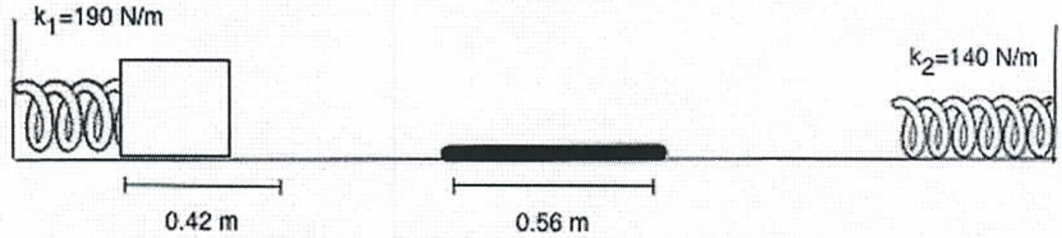
(b) At 31.0 s what is the speed of a point on the merry-go-round 2.50 m from the axis? (2 pts)

$V = \omega r = (1.22\,\frac{rad}{s})(2.50\,m) = \boxed{3.04\,\frac{m}{s}}$

(c) What was its angular acceleration? (4 pts)

$\alpha = \frac{\omega - \omega_0}{t} = \frac{1.22\,\frac{rad}{s}}{31.0\,s} = \boxed{0.039\,\frac{rad}{s^2}}$

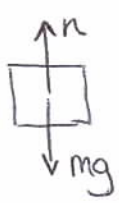
4. Two ideal springs are fixed at the ends of a long table. A 3.2 kg block is held compressing one spring with spring constant $k_1 = 190$ N/m a distance of 0.42 m. The surface is frictionless except for a rough patch 0.56 m in length with μ_k=0.21 positioned halfway between the springs. The second spring has constant $k_2 = 140$ N/m. When released from rest, the block slides across the surface, encounters the second spring, bounces back to the first spring, and repeats this process a few times.



a) What are the kinetic and potential energies of the block the moment it is released? (3 pts)

$K = \frac{1}{2}mv^2 = 0$ (at rest)

$U = \frac{1}{2}kx^2 = \frac{1}{2}(190\,\frac{N}{m})(0.42m)^2 = 16.76\,J \simeq \boxed{17\,J}$



b) Determine the work done by friction during one pass. (3 pts)

$|\vec{F}_k| = \mu_k n = \mu_k mg = 6.59$ N

$W = -F_k d = -(6.59\,N)(0.56\,m) = -3.69\,J \simeq \boxed{-3.7\,J}$

c) How far is the right spring compressed the first time? (3 pts)

$\Delta K + \Delta U = W_{nc}$

$K_f - K_i + U_f - U_i = W_f$

$0 - 0 + \frac{1}{2}k_f x^2 - 16.76J = -3.69J$

$\frac{1}{2}k_f x^2 = 14.07J$

$\rightarrow x_f = \sqrt{\frac{2}{k_f}(14.07J)} = \boxed{0.45\,m}$

d) How many times will the block make it past the rough patch? (6 pts)

On each pass, you lose 3.7 J due to friction.

Find how many passes until energy goes to zero.

$16.76J - (3.7J)n \overset{set}{=} 0$

$n = \frac{16.76J}{3.7J} = \boxed{4.54 \text{ times}}$

$\boxed{\text{4 times completely through.}}$

e) On the last pass, how far through the rough patch does the block travel before coming to a stop? (5 pts)

Find how much energy is lost after 4 times.

$16.76J - 4(3.7J) = 2.0J$ remaining.

See at what distance it takes friction to do -2.0J of work.

$-2.0J = -f_k d$

$d = \frac{-2.0J}{-6.59\,N} = \boxed{0.30\,m}$

5. A particle is trapped in a potential energy well where the potential energy can be expressed as

$$U = (4.2 + 5.1x + 2.3x^2)J$$

where U is in joules when x is in meters. The particle has total mechanical energy 12.4J.

a) Find the turning points of the particle. (7 pts)

particle: $K+U = 12.4J$; particle turns when $K=0 \Rightarrow U = 12.4J$

$12.4J \overset{set}{=} (4.2 + 5.1x + 2.3x^2)J$

$0 = (-8.2 + 5.1x + 2.3x^2)J$

use quadratic formula

$$x = \frac{-5.1 \pm \sqrt{(5.1)^2 - 4(2.3)(-8.2)}}{2(2.3)} = 1.07\,m, -3.29\,m \simeq \boxed{1.1m, -3.3m}$$

b) What is the force on the particle at each of the turning points? (4 pts)

$F = -\frac{dU}{dx} = (-5.1 - 4.6x)N$

$F = -(5.1 + 4.6(1.1m)) = \boxed{-10.2\ N}$

$F = -(5.1 + 4.6(-3.3m)) = \boxed{+10.1\ N}$

6. Perform the following calculations using the vectors listed below. (3 pts each)

$|\vec{A}| = \sqrt{(2.4)^2 + (5.6)^2 + (-4.9)^2}$

$= 7.82 \simeq 7.8$

$|\vec{D}| = \sqrt{(-6.2)^2 + (-3.6)^2 + (-5.2)^2}$

$= 8.86 \simeq 8.9$

$$\vec{A} = 2.4\hat{i} + 5.6\hat{j} - 4.9\hat{k}$$

$$\vec{B} = 2.8\hat{i} - 1.2\hat{j}$$

$$\vec{C} = 9.0\hat{i} + 2.4\hat{j} + 5.2\hat{k}$$

$$\vec{D} = \vec{B} - \vec{C}$$

$\vec{D} = (2.8 - 9.0)\hat{i} + (-1.2 - 2.4)\hat{j} + (0 - 5.2)\hat{k} = -6.2\hat{i} - 3.6\hat{j} - 5.2\hat{k}$

a) Calculate $\vec{A} \cdot \vec{B}$ and find the angle between the two vectors.

$\vec{A} \cdot \vec{B} = (2.4)(2.8) + (5.6)(-1.2) + (-4.9)(0)$

$= 6.72 - 6.72 = \boxed{0}$

$\vec{A} \cdot \vec{B} = AB\cos\theta$

$0 = AB\cos\theta \Rightarrow \cos\theta = 0 \Rightarrow$ $\boxed{\theta = \frac{\pi}{2} \text{ or } 90°}$

b) Calculate $\vec{A} \cdot \vec{D}$ and find the angle between the two vectors.

$\vec{A} \cdot \vec{D} = (2.4)(-6.2) + (5.6)(-3.6) + (-4.9)(-5.2)$

$= -14.88 + (-20.16) + 25.48 = \boxed{-9.56 = \vec{A} \cdot \vec{D}}$

$\vec{A} \cdot \vec{D} = AD\cos\theta$

$\cos\theta = \frac{-9.56}{(7.82)(8.86)} = -0.138 \Rightarrow \boxed{\theta = 1.7 \text{ radians or } 98°}$

c) Calculate $(\vec{A} \cdot \vec{C}) + (\vec{B} \cdot \vec{D})$.

$\vec{A} \cdot \vec{C} = (2.4)(9.0) + (5.6)(2.4) + (-4.9)(5.2)$

$= 9.56$

$\vec{B} \cdot \vec{D} = (2.8)(-6.2) + (-1.2)(-3.6) + (0)(-5.2)$

$= -13.04$

$9.56 + (-13.04) = -3.48 \simeq \boxed{-3.5}$

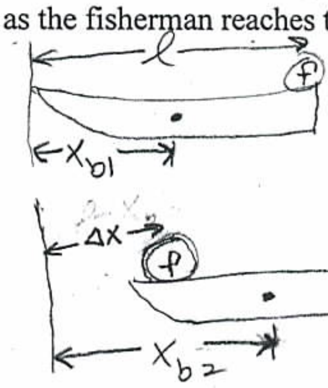
7. A 100.-kg fisherman maneuvers his 125-kg boat so the prow is just touching a large rock. He then leaves his seat at the rear of the boat and walks the 4.00 m relative to the boat to the prow, from which he hopes to step out onto the rock. The friction between the boat and the water can be ignored.

(a) Describe the motion of the center-of-mass of the fisherman-plus-boat system as the fisherman walks from one end of the boat to the other. Explain your reasoning. (4 pts)

The CM remains at rest, since $\vec{F}_{net\,ext} = M\vec{a}_{CM} = 0$, i.e., there is no net external force on the system.

(b) Calculate the distance moved by the boat relative to the rock as the fisherman reaches the prow. (6 pts)

$$X_{CM1} = X_{CM2} = X_{CM}$$



$$X_{CM} = \frac{m_f \ell + m_b X_{b1}}{m_f + m_b} = \frac{m_f(\Delta x) + m_b X_{b2}}{m_f + m_b}$$

$$m_f \ell = m_b(X_{b2} - X_{b1}) + m_f(\Delta x)$$

$$\Delta x = (X_{b2} - X_{b1}),\ \text{so}$$

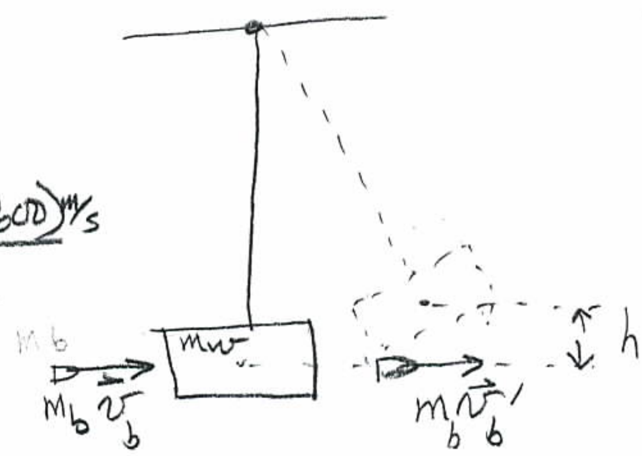
$$\Delta x = \frac{m_f \ell}{m_b + m_f} = \frac{(100.\,kg)(4.00\,m)}{(125\,kg) + 100.\,kg} = \underline{1.78\,m}$$

8. A 10.0-g bullet is fired horizontally at a speed of 750. m/s into a 500.-g block of wood that is hanging by a string of length 1.00 m. The bullet goes through the block, exiting with a horizontal speed of 600. m/s. Calculate the maximum vertical displacement of the block as a result of the collision. (10 pts)

Collision $\Rightarrow \vec{P}_i = \vec{P}_f$

$$m_b v_b = m_b v_b' + m_w v_w$$

$$v_w = \frac{m_b(v_b - v_b')}{m_w} = \frac{0.01\,kg\,(750 - 600)\,m/s}{0.5\,kg}$$



$v_w = 3.00$ m/s

After collision

$K_{iw} + U_{iw} = K_{fw} + U_{fw}$ for wood alone (only $M\vec{g}$ does work)

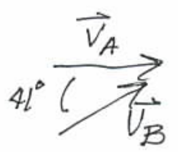
$$\tfrac{1}{2} m_w v_w^2 + 0 = 0 + m_w g h$$

$$h = \frac{v_w^2}{2g} = \frac{(3.\,m/s)^2}{2(9.8\,m/s^2)} = \underline{0.459\,m}$$

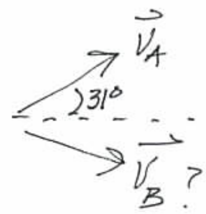
9. Object A has a mass of 8.7 kg and moves with a speed of 5.6 m/s in the positive-x direction. Object B has a mass of 7.4 kg and moves with a speed of 7.2 m/s so that $\vec{v_B}$ makes an angle of 41° with $\vec{v_A}$. The two objects collide; after the collision object A has a speed of 5.5 m/s and has been deflected 31° from its original direction.

(a) Sketch the system of object A and object B both before and after the collision. Indicate the directions of each velocity vector on your sketches and identify any known angles. (2 pts)

Before



After



(b) Find the velocity of object B after the collision. (8 pts)

Momentum is conserved!

$$(8.7\,kg)(5.6\,m/s\ \hat{i}) + (7.4\,kg)[7.2\,m/s\cos 41^\circ\,\hat{i} + 7.2\,m/s\sin 41^\circ\,\hat{j}]$$
$$= (8.7\,kg)[5.5\,m/s\cos 31^\circ\,\hat{i} + 5.5\,m/s\sin 31^\circ\,\hat{j}] + (7.4\,kg)\,\vec{v}_B$$

$$\Rightarrow \vec{v}_B = [6.5\,\hat{i} + 1.4\,\hat{j}]\ m/s$$

(c) Determine whether this collision is elastic or inelastic (3 pts)

Does $K_i = K_f$?

$$K_i = \tfrac{1}{2}(8.7\,kg)(5.6\,m/s)^2 + \tfrac{1}{2}(7.4\,kg)(7.2\,m/s)^2 = 330\,J$$
$$K_f = \tfrac{1}{2}(8.7\,kg)(5.5\,m/s)^2 + \tfrac{1}{2}(7.4\,kg)[(6.5\,m/s)^2 + (1.4\,m/s)^2] = 290\,J$$

$K_i \neq K_f$, so inelastic

10. In each of the four situations shown in the figure below, an object explodes into two equal mass fragments when the object is at the origin of the coordinate system. The velocity vectors of the fragments are indicated; in each case they are directed either along an axis or at 45° to an axis. For each situation, indicate on the graph the direction of travel of the object before the explosion (or put a 0 on the graph if the object was initially at rest). (4 pts)