PHYSICS 2110
Final Exam

December 11, 2001

Name (please print)______________________________ Seat Number ____________

Student ID Number ______________________________ Class meeting time: ________

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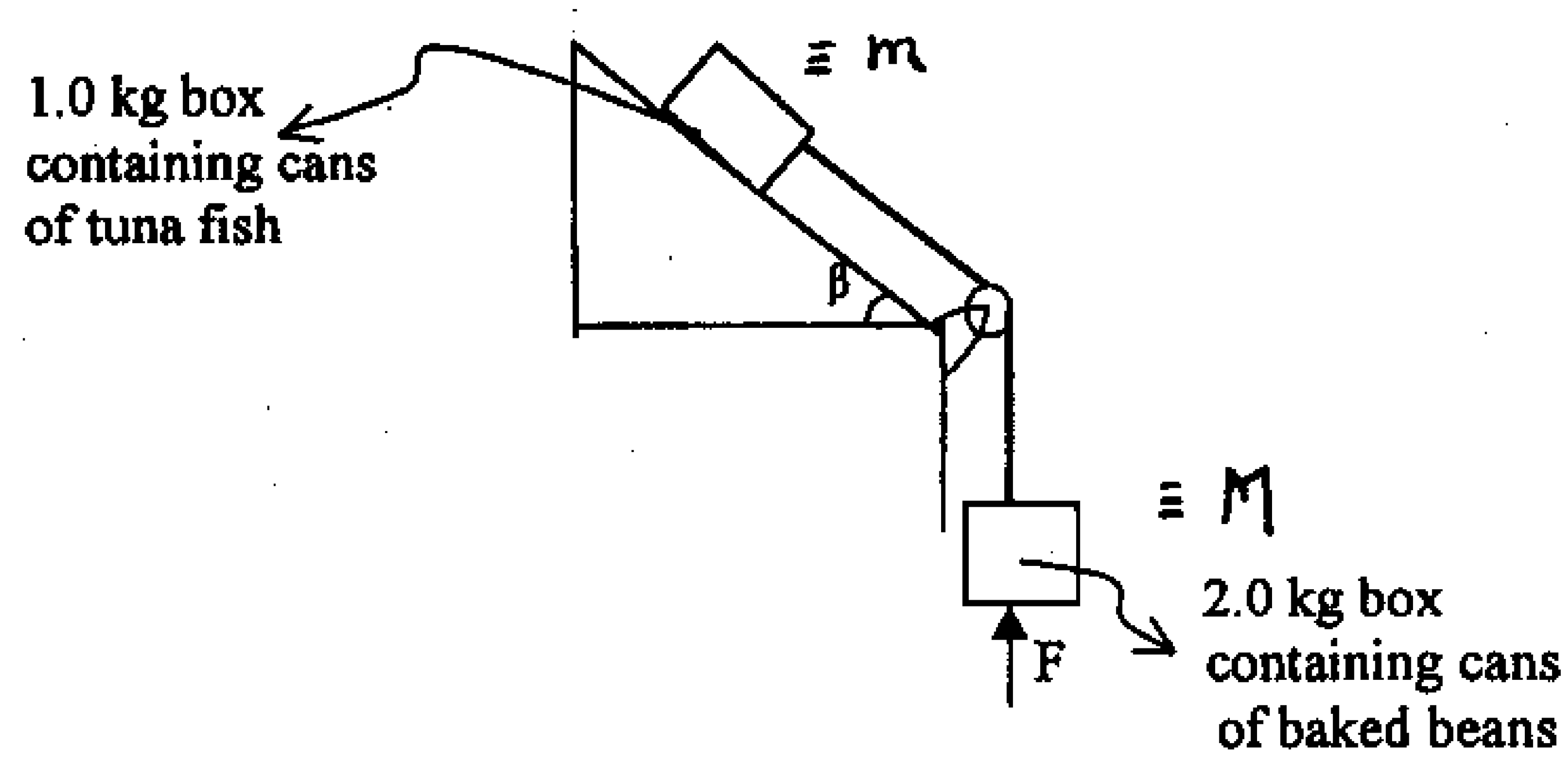
REMEMBER: *YOU MUST SHOW YOUR WORK AND/OR EXPLAIN YOUR REASONING TO RECEIVE CREDIT! Write down given quantities and identify the quantities requested in the problem.* ***Draw pictures***—*they will help you visualize the system.* ***Write down the formula(s)*** *you are using and, again,* ***show your work!*** *Finally, in numerical results,* ***don't forget the units*** *and* ***watch the significant figures!***

$g = 9.80\ m/s^2$ $c_{water} = 1.00$ cal/g-K=4190 J/kg-K $v = 343$ m/s (speed of sound)
$I_0 = 10^{-12}\ W/m^2$ $R = 8.314$ J/mol-K = 0.0821 L-atm/mol-K
1 atm = $1.01 \times 10^5\ N/m^2$ = 101 kPa

QUESTION NUMBER	POSSIBLE SCORE	YOUR SCORE
1	15	________
2	15	________
3	10	________
4	13	________
5	12	________
6	8	________
7	7	________
8	10	________
9	10	________
	TOTAL	________

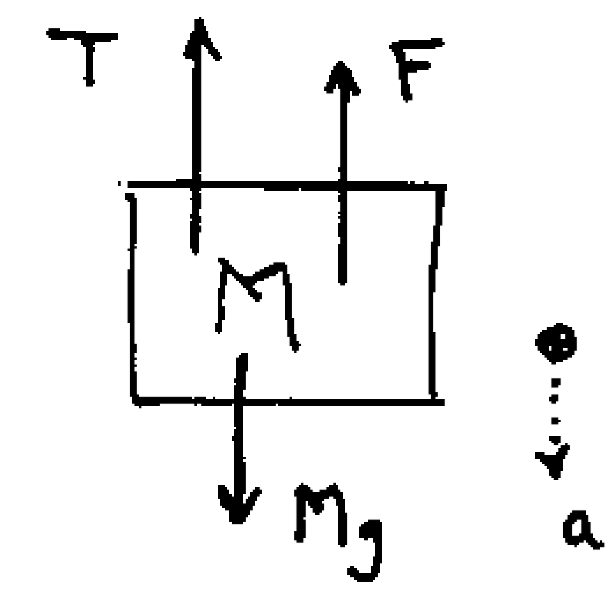
1.



In the figure above, a 1.0 kg box of tuna fish cans on a frictionless inclined surface is connected to a 2.0 kg box of baked beans. The pulley is massless and frictionless. An upward force F of 6.0 N acts on the baked beans case, which has a downward acceleration of 5.5 m/s^2.

a) What is the tension in the connecting cord? (10 points)

Draw the Free-Body Diagram for the hanging mass. Adding up the downward forces, Newton's 2^{nd} Law gives



$$Mg - T - F = Ma$$

Solve for T:

$$T = Mg - Ma - F$$

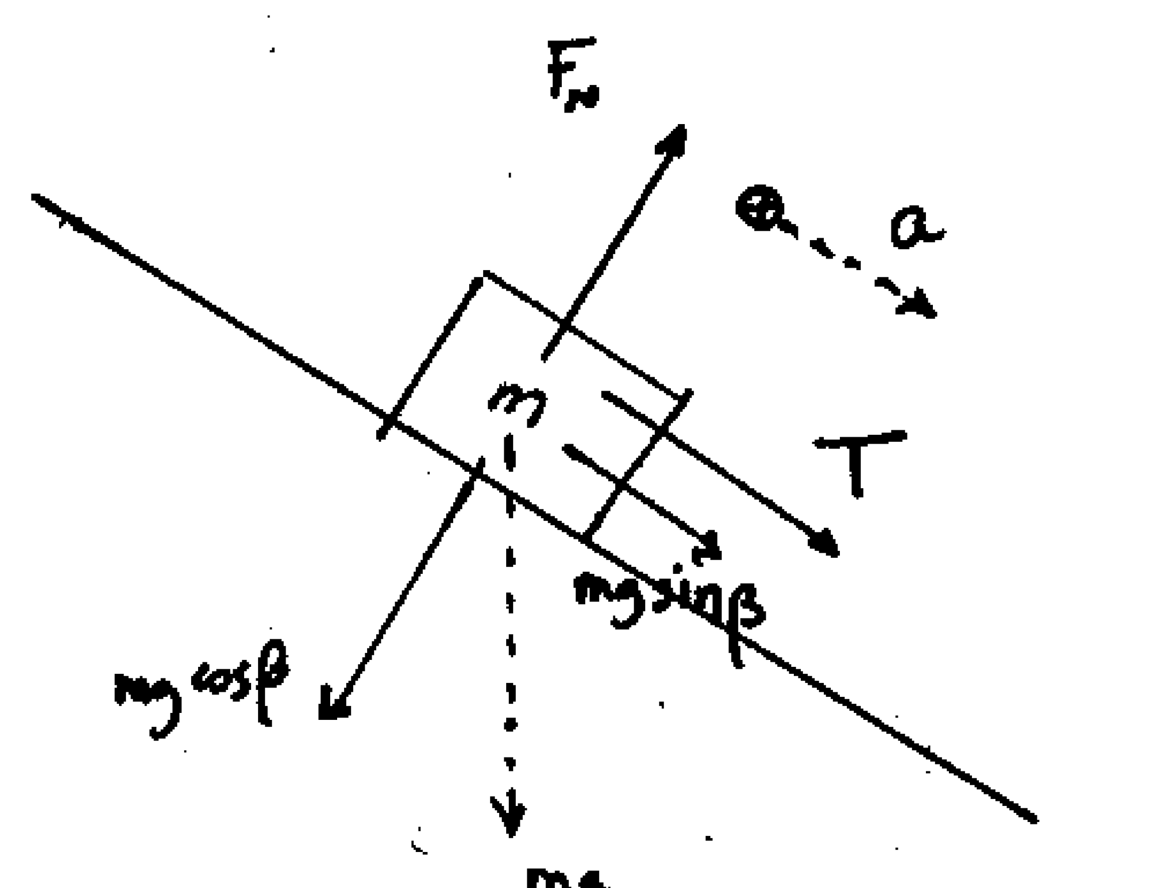
$$= (2.0\ kg)(9.80\ m/s^2) - (2.0\ kg)(5.5\ m/s^2) - 6.0\ N$$

$$= \boxed{2.6\ N}$$

F = 6.0 N
M = 2.0 kg
a = 5.5 m/s^2

b) What is the angle β? (5 points)

Draw the Free-Body Diagram for the mass on the slope. (Note: No friction; decompose force of gravity mg as usual.) Adding up the forces acting down the slope, Newton's 2^{nd} Law gives



$$T + mg \sin\beta = ma$$

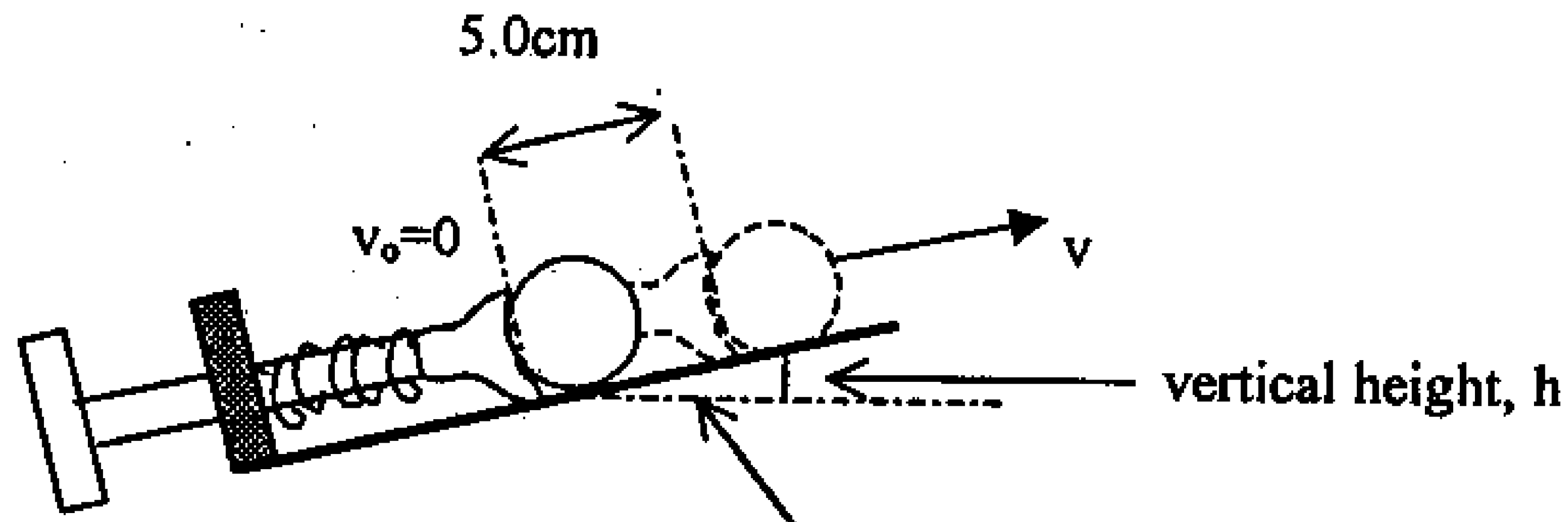
m = 1.0 kg
T = 2.6 N
a = 5.5 m/s^2

Solve for β:

$$\sin\beta = \frac{ma - T}{mg} = \frac{(1.0\ kg)(5.5\ m/s^2) - 2.6\ N}{(1.0\ kg)(9.80\ m/s^2)} = 0.296$$

$$\rightarrow \boxed{\beta = 17°}$$

2.



The game board is inclined 8.5° above the horizontal

A pinball machine launches a 100. g ball with a spring driven plunger. The game board is inclined at 8.5 ° above the horizontal. Find the spring constant k of the spring that will give the ball a speed of 83 cm/s, when the plunger is released from rest with the spring compressed 5.0 cm from its relaxed position. Assume that the plunger's mass and frictional effects are negligible. The dashed lines in the figure above shows the position of the ball when it has the maximum speed just as it loses contact with the plunger. (15 points)

Meas'ing height from the initial position, the initial mech. energy is just the energy stored in the spring. With $x = 5.0\text{ cm} = 0.050\text{ m}$,

$$E_i = \frac{1}{2}kx^2$$

In the final position the ball has kinetic energy $\frac{1}{2}mv^2$ and also grav. pot'l energy mgh where $h = x \sin 8.5°$. Thus:

$$E_f = \frac{1}{2}mv^2 + mg\,x \sin 8.5°$$

With no friction forces, energy is conserved: $E_i = E_f$. Solve for k:

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + mgx \sin 8.5°$$

$$k = \frac{mv^2 + 2mgx\sin 8.5°}{x^2} = \frac{(0.100\text{ kg})(0.83\text{ m/s})^2 + 2(0.100\text{ kg})(9.80\text{ m/s}^2)(0.050\text{ m})\sin 8.5°}{(0.050\text{ m})^2}$$

$$= \boxed{33.4\ \text{N/m}}$$

3. The turntable of a record player rotates initially at a rate of 33.0 revolutions /min and takes 20.5 s to come to rest.

a) What is the angular acceleration (in rad/s^2) of the turntable, assuming the acceleration is uniform? (3 points)

$$\omega_o = 33.0\frac{\text{rev}}{\text{min}}\cdot\frac{2\pi\text{ rad}}{\text{rev}}\cdot\frac{1\text{ min}}{60\text{ s}} = 3.46\text{ rad/s}$$

$$\alpha = \frac{\omega-\omega_o}{t} = \frac{0-34.6\text{ rad/s}}{20.5\text{ s}} = \boxed{0.169\text{ rad/s}^2}$$

b) How many rotations does the turntable make before coming to rest? (3 points)

We can use:

$$\theta = \frac{1}{2}(\omega_o+\omega)t = \frac{1}{2}(34.6\text{ rad/s} + 0)(20.5\text{ s})$$

$$= 35.4\text{ rad}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{5.64\text{ rev}}$$

c) If the radius of the turntable is 14 cm, what is the initial linear speed of a point on the rim of the turntable? (1 point)

Initial value of v: (at $r = 0.14$ m)

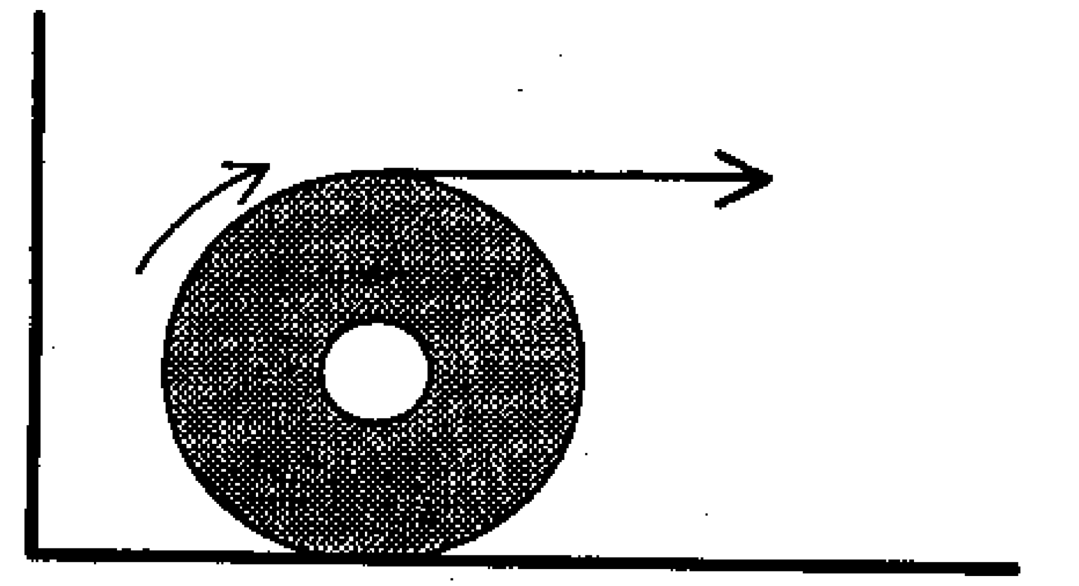
$$v = \omega_o r = (34.6/\text{s})(0.14\text{ m}) = \boxed{0.48\text{ m/s}}$$

d) What are the magnitudes of the radial and tangential components of the linear acceleration of the point on the rim at t=0? (3 points) At $t=0$, $\omega = 3.46$ rad/s

$$a_c = \frac{v^2}{r} = \omega^2 r = (3.46\text{ rad/s})^2(0.14\text{ m}) = \boxed{1.67\text{ m/s}^2}$$

$$a_T = \alpha r = (0.169\text{ rad/s}^2)(0.14\text{ m}) = \boxed{0.024\text{ m/s}^2}$$

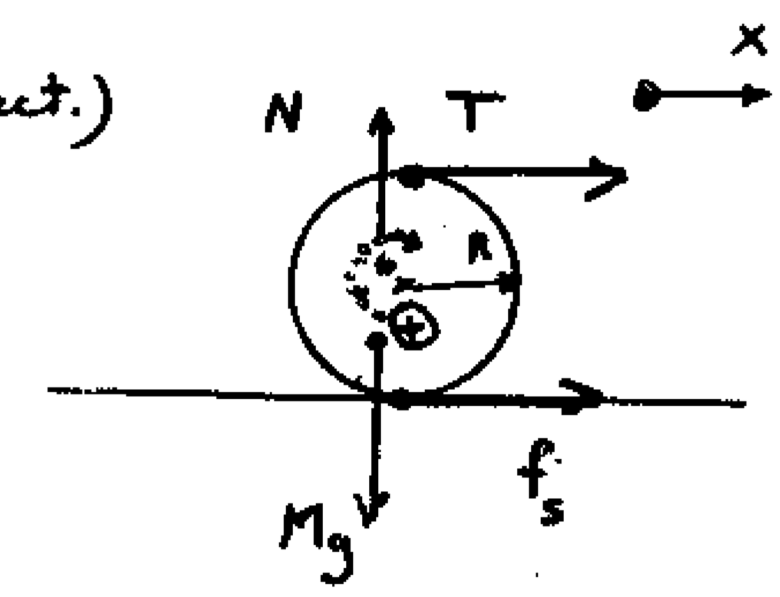
4. A puppy pulls paper off the top of a roll, which rolls without slipping on a horizontal surface (to the right in the diagram). The center of mass of the roll accelerates at 1.20 m/s^2. The outer radius of the roll is 6.00 cm, its mass is 0.400 kg, and its moment of inertia is 0.000576 kg-m^2. The thickness and mass of the paper removed from the roll are negligible.



a) If she starts at the position of the roll, how far must the puppy travel to unwind 3.00 m of paper? (4 points) If a length $l = 3.00$ m of the paper unwinds then a point on the roll (i.e. the dog!) has moved by $+l$ relative to the roll. However since the roll turns thru an angle $\theta = l/r$ then the roll has rolled a distance $x = \theta \cdot r = \frac{l}{r} \cdot r = l$ relative to the floor. Adding these displacements, the dog has moved a distance $l + l = 2l = 2(3.0\text{ m}) = \boxed{6.0\text{ m}}$ relative to the floor

b) Calculate the minimum coefficient of friction needed for pure rolling. (9 points)

First, draw a Free-Body-Diagram, as shown at the right.
(Choose dir of fric force as forward; sign of f_s tells us if this is correct.)



Newton's 2nd Law for x-direction gives

$$T + f_s = Ma \quad (1)$$

Newton's 2nd Law in Rot'l form gives (clockwise = +):

$$TR - f_s R = I\alpha = Ia/R$$

Divide by R, get

$$T - f_s = Ia/R^2 \quad (2)$$

Now subtract (2) from (1). Get:

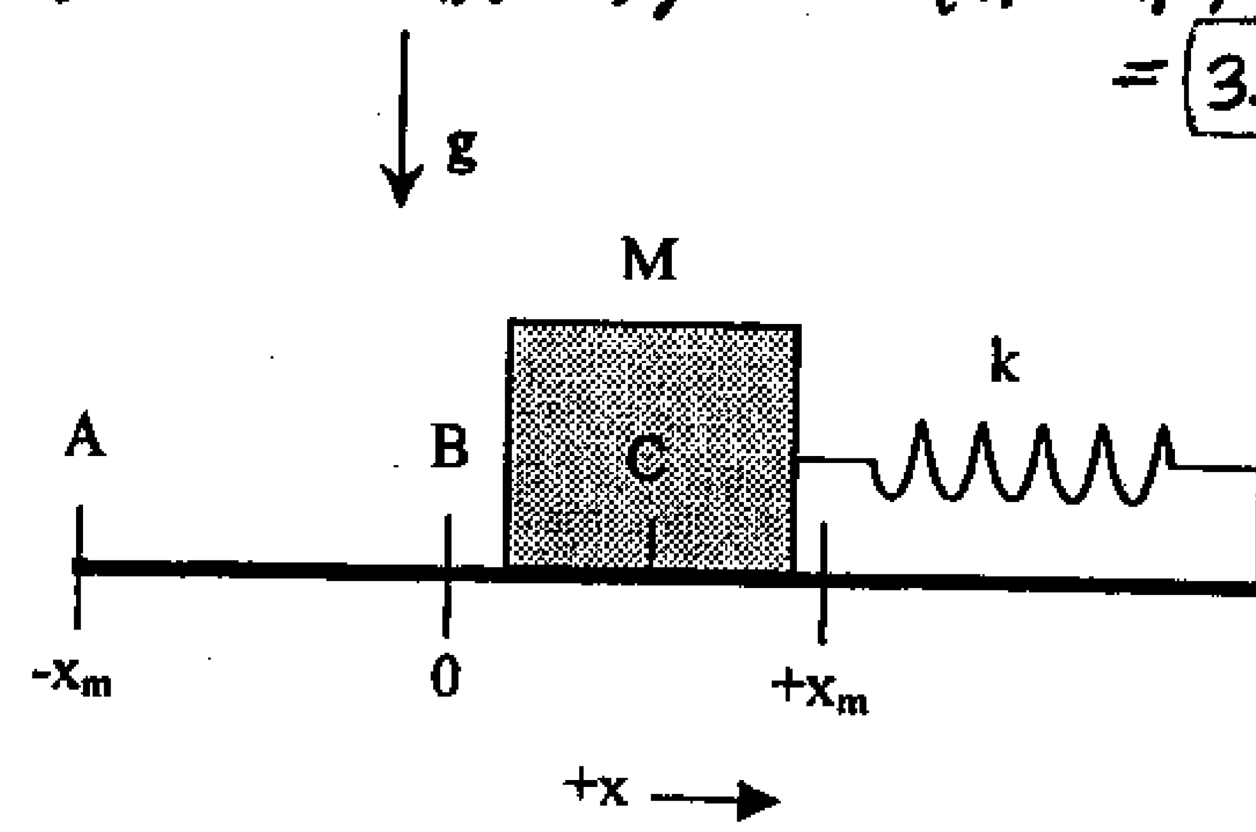
$$2f_s = a\left(M - I/R^2\right)$$

But for minimum possible μ_s to roll without slipping, $f_s = \mu_s N$ and since $N = Mg$, $f_s = \mu_s Mg$. This gives:

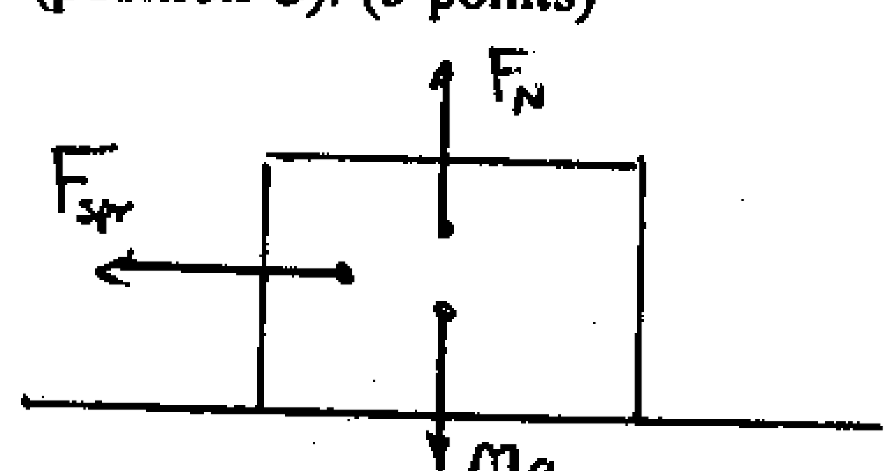
$$2\mu_s Mg = a\left(M - I/R^2\right)$$

$$\mu_s = \frac{a}{2Mg}\left(M - \frac{I}{R^2}\right) = \frac{(1.20\text{ m/s}^2)}{2(0.400\text{ kg})(9.80\text{ m/s}^2)}\left(0.400\text{ kg} - \frac{5.76\times10^{-4}\text{ kg m}^2}{(0.0600\text{ m})^2}\right) = 0.0367 = \boxed{3.67\times10^{-2}}$$

5. A block of mass M=3.00 kg is attached to an ideal spring of stiffness constant k=66.0 N/m and oscillates on a smooth horizontal surface between $-x_m$ and $+x_m$ as shown. The total energy of the block is 15.0 J.



a) Draw a neat and properly labeled free-body diagram for the mass in the position shown (position C). (3 points)



Mg = weight of block
F_N = normal force of surface
F_{spr} = force of spring; must push to left because spring is compressed

b) Complete the following statements with the letter(s) corresponding to position(s) A, B, or/and C. If none of these positions correctly satisfies the statement, write "none" in the blank. (6 points)

The velocity of the block is zero at position(s) ___A___.
The maximum velocity of the block occurs at position(s) ___B___.
The acceleration of the block is zero at position(s) ___B___.
The maximum acceleration of the block occurs at position(s) ___A___.
The acceleration is positive at position(s) ___A___.
The acceleration is negative at position(s) ___C___.

c) Calculate the amplitude of the motion. (3 points)

Since the total energy of the motion is $E_{tot} = \frac{1}{2} k x_m^2$, solve for x_m:

$$x_m^2 = 2E_{tot}/k = 2(15.0\text{ J})/(66.0\text{ N/m}) = 0.455\text{ m}^2 \qquad \boxed{x = 0.674\text{ m}}$$

6. A traveling wave in a string is given by

$$y(x,t) = (0.0150m)\cos[(40.0rad/s)t + (1.20rad/m)x].$$

Determine each of the following: (2 points each)

a) the wave speed.

Here, $k = (1.20/m)$ and $\omega = (40.0/s)$

Coeff's of x and t in arg of cosine

Then use:

v = wave speed $= \omega/k = \frac{(40.0/s)}{(1.20/m)} = \boxed{33.3\ \frac{m}{s}}$

b) the direction of propagation.

Here the relative + sign between the x and t terms in the argument of the oscillatory term tells us that the wave travels in the $\boxed{-x \text{ direction.}}$

For the phase $kx + \omega t$ to be constant, if t increases x must decrease! $\Rightarrow -x$ direction.

c) the wavelength.

Since $k = 2\pi/\lambda$ we have:

$\lambda = 2\pi/k = \frac{2\pi}{(1.20/m)} = \boxed{5.24\ m}$

d) the velocity of the point on the string at x=2.00 m at time t=1.50 s.

Velocity of the points of the string is given by

$v_y(x,t) = \frac{\partial y}{\partial t} = -(0.0150\,m)(40.0/s)\sin[(40.0/s)t + (1.20/m)x]$

Evaluate at $x = 2.00\,m$, $t = 1.50\,s$ (remember to use radians mode on calc!)

Get: $v_y(2.0m, 1.50s) = \boxed{+0.251\ m/s}$

7. An electrical wire having a linear density of 0.100 kg/m is attached to two poles 8.00 m apart. A small branch falling from a tree strikes the wire, causing it to vibrate in its second harmonic frequency of 20.0 Hz.

a) Calculate the tension in the wire. (4 points)

$\mu = 0.100\ kg/m$ $L = 8.00\,m$, length of wire

For a cord vibrating in the second harmonic,

$L = \lambda = 8.00\,m$ so that the speed of waves on the wire is

$v = \lambda f = (8.00\,m)(20.0/s) = 160.\ m/s$

Then, from $v = \sqrt{\frac{\tau}{\mu}}$, solve for τ:

$v^2 = \frac{\tau}{\mu}$ $\tau = \mu v = (0.100\frac{kg}{m})(160.\frac{m}{s})^2 = \boxed{2.56\times10^3\,N}$

b) Approximately where, relative to the ends or/and center of the wire, did the branch hit the wire? (3 points)

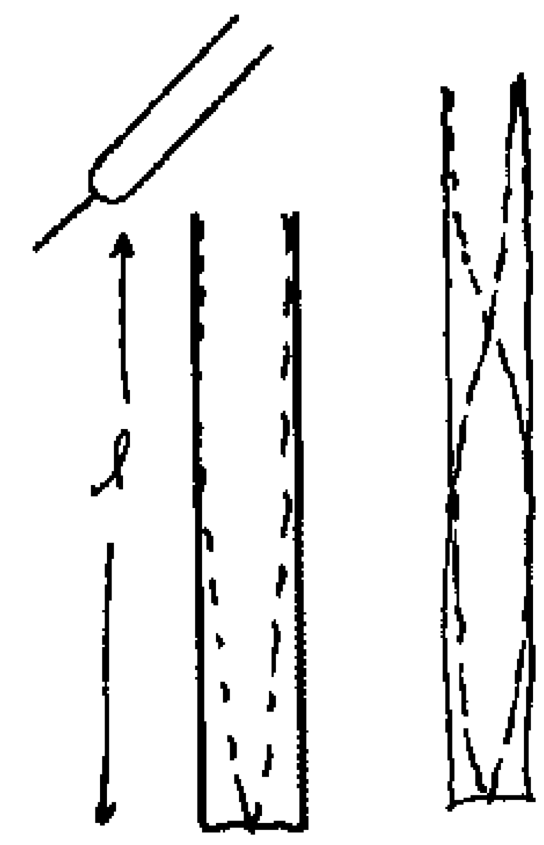
The branch would have to give a large displacement to a position where the 2^{nd} harmonic has large displacements, i.e. $\boxed{\text{1/4 of the way}}$ from either end. (Actually, such an initial disturbance on the wire would also set the 1^{st} harmonic into motion simultaneously.)

8. **Sound Bytes:**

a) A sound wave is a longitudinal wave. Tell what is meant by the term *longitudinal wave*. (2 points)

A longitudinal wave is one for which the (small) displacements of the medium are parallel to the direction of wave motion.

b) A cylindrical tube is open on one end; one can adjust the height of the air column by lowering the bottom level. It is found that for a sound of a given frequency, the first (fundamental) resonance occurs when the column has a height of 20.0 cm. If we lower the bottom level until the very next resonance occurs, what is the new height of the air column? (3 points)



The first resonance occurs when $\ell = \lambda/4$, so $\lambda/4 = 20.0$ cm. The next resonance occurs when $\ell = \frac{3\lambda}{4}$ (λ stays the same) so $\ell = 3\cdot(\frac{\lambda}{4}) = 3(20.0 \text{ cm})$

$= \boxed{60.0 \text{ cm}}$ for the new height.

c) An ambulance whose siren plays a 600. Hz sound is driving directly toward a stationary listener; the listener hears a frequency of 650. Hz. What is the speed of the ambulance? [Use $343\frac{m}{s}$ as the speed of sound.] (5 points)

Here the detector is stationary ($v_D = 0$) and the source is moving toward the detector so that a higher frequency is heard. Choosing the correct sign in the "Doppler formula" we have

$$f' = 650 \text{ Hz} = \left(\frac{v}{v - v_s}\right) f = \left(\frac{1}{1 - v_s/v}\right)(600 \text{ Hz})$$

Solve for v_s:

$$\left(1 - \frac{v_s}{v}\right) = \frac{600}{650} = 0.923$$

$$\rightarrow \frac{v_s}{v} = 1 - 0.923 = 7.69\times 10^{-2}$$

$$v_s = (7.69\times 10^{-2})(343\tfrac{m}{s}) = \boxed{26.4 \tfrac{m}{s}}$$

9. **Thermal Quickies:**

a) 100.0 grams of an unknown metal at 100.0°C is immersed in 200.0 g of water at 20.0°C. When at equilibrium, both have a final temperature of 22.2°C. Ignore any losses of heat to the container or the surroundings. What is the specific heat of the unknown metal? [Specific heat of water is $4190 \frac{J}{kg\cdot K}$.] (5 points)

$\Delta T_{met} = 22.2°C - 100.0°C$
$= -77.8$ K

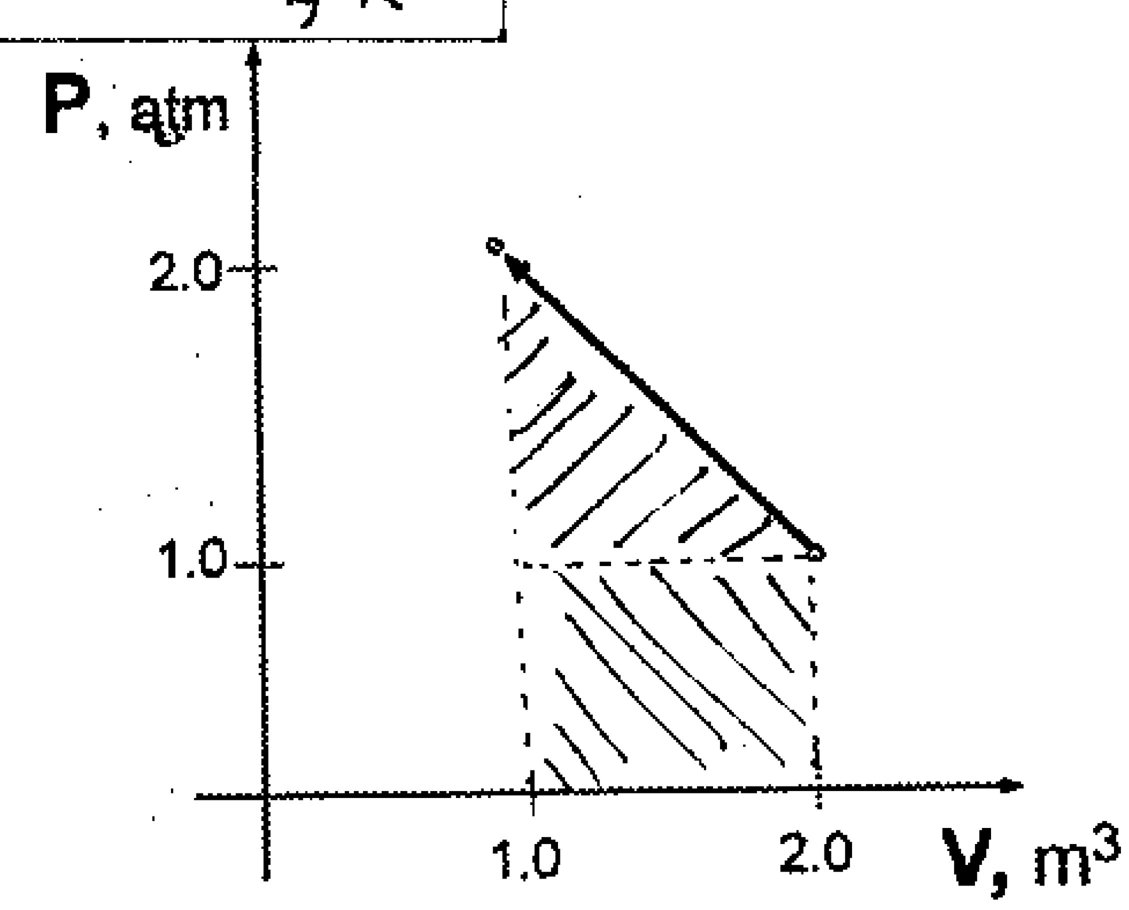
Total heat gain of components is zero:

$$m_w c_w \Delta T_w + m_{met} c_{met} \Delta T_{met} = 0$$

$$(0.200 \text{ kg})(4190 \tfrac{J}{kg\,K})(+2.2 \text{ K}) + (0.100 \text{ kg})\, c_{met} (-77.8 \text{ K}) = 0$$

Solve for c_{met}. Gives: $\boxed{c_{met} = 237 \frac{J}{kg\cdot K}}$

b) A gas undergoes the compression shown at the right: Starting at 1.00 atm of pressure and a volume of $2.00\,m^3$, it is compressed to 2.00 atm of pressure and a volume of $1.00\,m^3$, following a straight-line path on the P-V diagram. Find the work *W* done by the gas. Express the answer in joules. (5 points)



$W = \int P\, dV$

Absolute value of area under curve is

$$(1 \text{ atm})(1 \text{ m}^3) + \tfrac{1}{2}(1 \text{ atm})(1 \text{ m}^3)$$

$$= 1.5 \text{ atm}\cdot\text{m}^3 = (1.5)(1.01\times 10^5 \tfrac{N}{m^2})\,\text{m}^3 = 1.5\times 10^5 \text{ J}$$

But path is a compression of the gas so W is negative: $\boxed{W = -1.5\times 10^5 \text{ J}}$