

PHYSICS 2110
Exam II

November 1, 2001

Name (please print) _____

Seat Number _____

Student ID Number _____

Class meeting time: _____

INSTRUCTOR (circle one)

Nesaraja

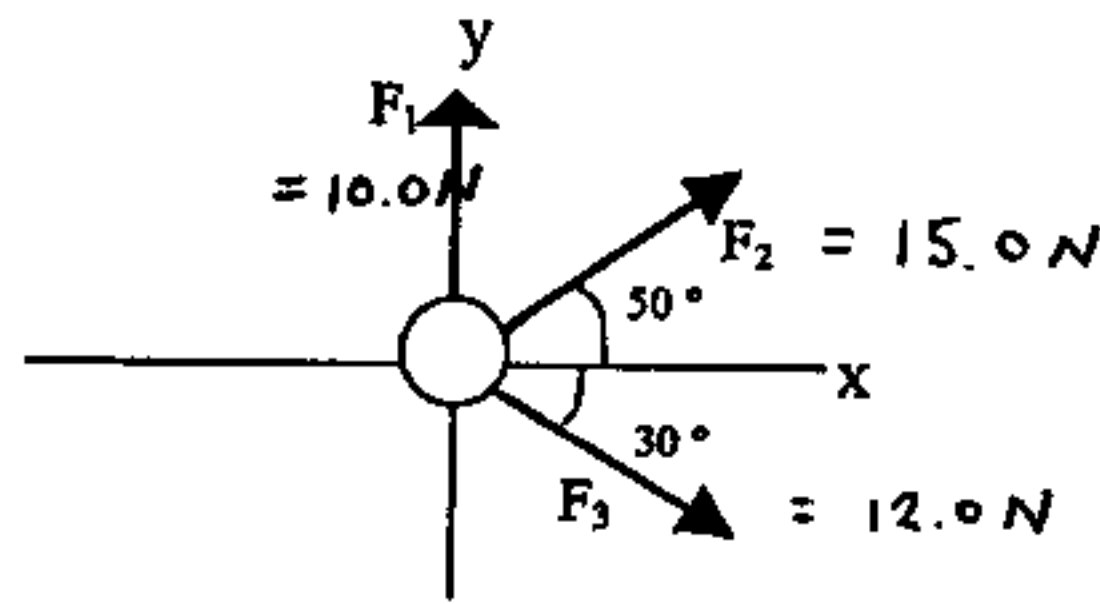
Murdock

Kozub

REMEMBER: YOU MUST SHOW YOUR WORK AND/OR EXPLAIN YOUR REASONING TO RECEIVE CREDIT! Write down given quantities and identify the quantities requested in the problem. Draw pictures—they will help you visualize the system. Write down the formula(s) you are using and, again, show your work! Finally, in numerical results, don't forget the units and watch the significant figures!

QUESTION NUMBER	POSSIBLE SCORE	YOUR SCORE
1	8	_____
2	16	_____
3	16	_____
4	33	_____
5	7	_____
6	14	_____
7	6	_____
	TOTAL	_____

1. The figure below shows an overhead view of a puck on a frictionless horizontal surface. Three constant horizontal forces act on the puck in the directions indicated. The magnitude of F_1 is 10.0 N, that of F_2 is 15.0 N and that of F_3 is 12.0 N.



- a) What is the magnitude and angle of F_{net} , the total force on the puck? (4 points)

Add x and y components to get \vec{F}_{net} :

$$F_{net, x} = 0 + (15.0 \text{ N}) \cos 50^\circ + (12.0 \text{ N}) \cos 30^\circ = 20.0 \text{ N}$$

$$F_{net, y} = 10.0 \text{ N} + (15.0 \text{ N}) \sin 50^\circ - (12.0 \text{ N}) \sin 30^\circ = 15.5 \text{ N}$$

We get: $|\vec{F}_{net}| = \sqrt{F_{net, x}^2 + F_{net, y}^2} = 25.3 \text{ N}$

Direction of $\vec{F}_{net} = \tan^{-1}\left(\frac{F_{net, y}}{F_{net, x}}\right) = 37.7^\circ$

- b) Since, the puck starts from rest and moves in the direction of the net force, find the net work W done on the puck by the three forces when the puck has gone through a displacement of magnitude $d=0.400 \text{ m}$. (4 points)

Here \vec{F}_{net} is parallel to the displacement, so

$$W = F \cdot d \cdot \underbrace{\cos \phi}_{=1} = (25.3 \text{ N})(0.400 \text{ m}) = 10.1 \text{ J}$$

2. An elevator has a mass of 1250 kg and carries a load of 845 kg. A constant frictional force of 4250 N retards its motion upward.

- a) What must be the minimum power delivered by the motor to lift the elevator at a constant speed of 3.00 m/s? (8 points)

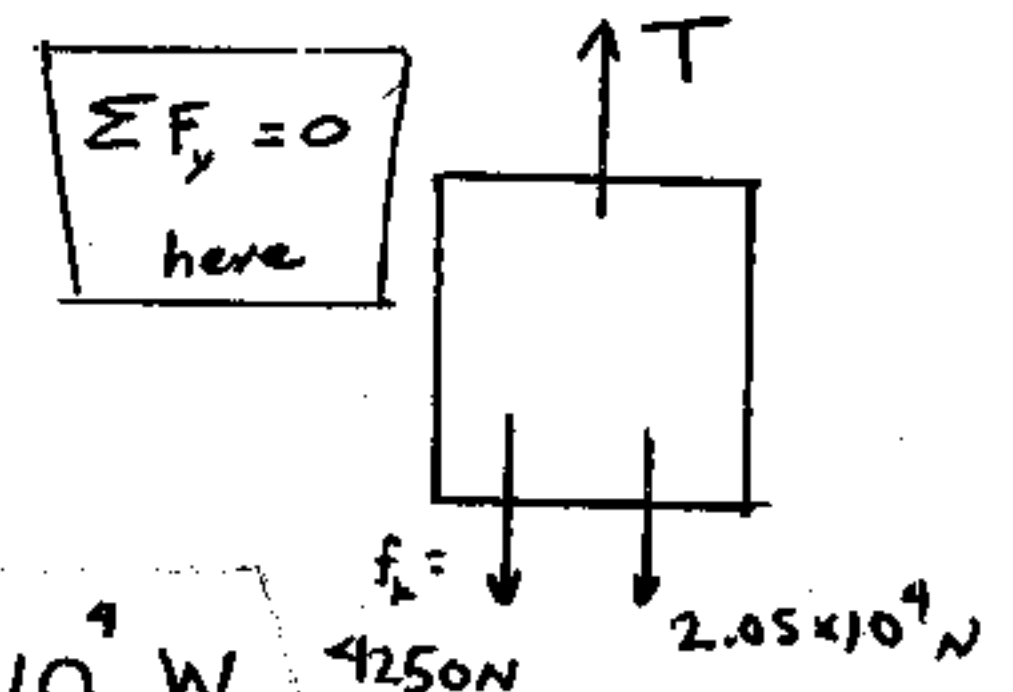
Total weight of elevator + load is

$$W = (1250 \text{ kg} + 845 \text{ kg})(9.8 \text{ m/s}^2) = 2.05 \times 10^4 \text{ N}$$

To lift at constant speed, tension in cable is

$$T = (2.05 \times 10^4 \text{ N} + 4250 \text{ N}) = 2.48 \times 10^4 \text{ N}$$

Power del'd is $P = Fv = (2.48 \times 10^4 \text{ N})(3.00 \text{ m/s}) = 7.43 \times 10^4 \text{ W}$



- b) If the motor is designed to provide an upward acceleration of 1.00 m/s^2 , what power must the motor deliver at the instant the speed is 5.00 m/s? (8 points)

Now the net force on elevator + contents is not zero, rather

$$\sum F_y = ma = (1250 \text{ kg} + 845 \text{ kg})(1.00 \text{ m/s}^2) = 2.10 \times 10^3 \text{ N}$$

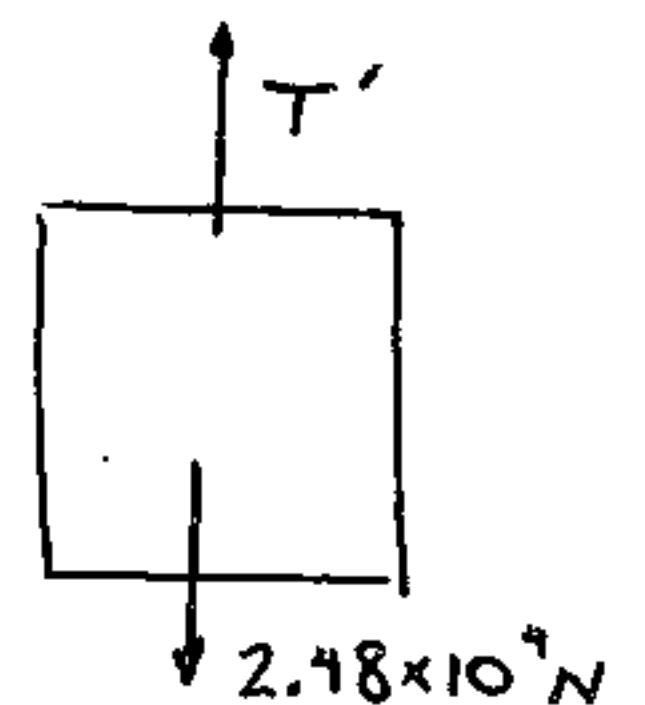
Now the tension in cable is

$$T' - (2.48 \times 10^4 \text{ N}) = 2.10 \times 10^3 \text{ N}$$

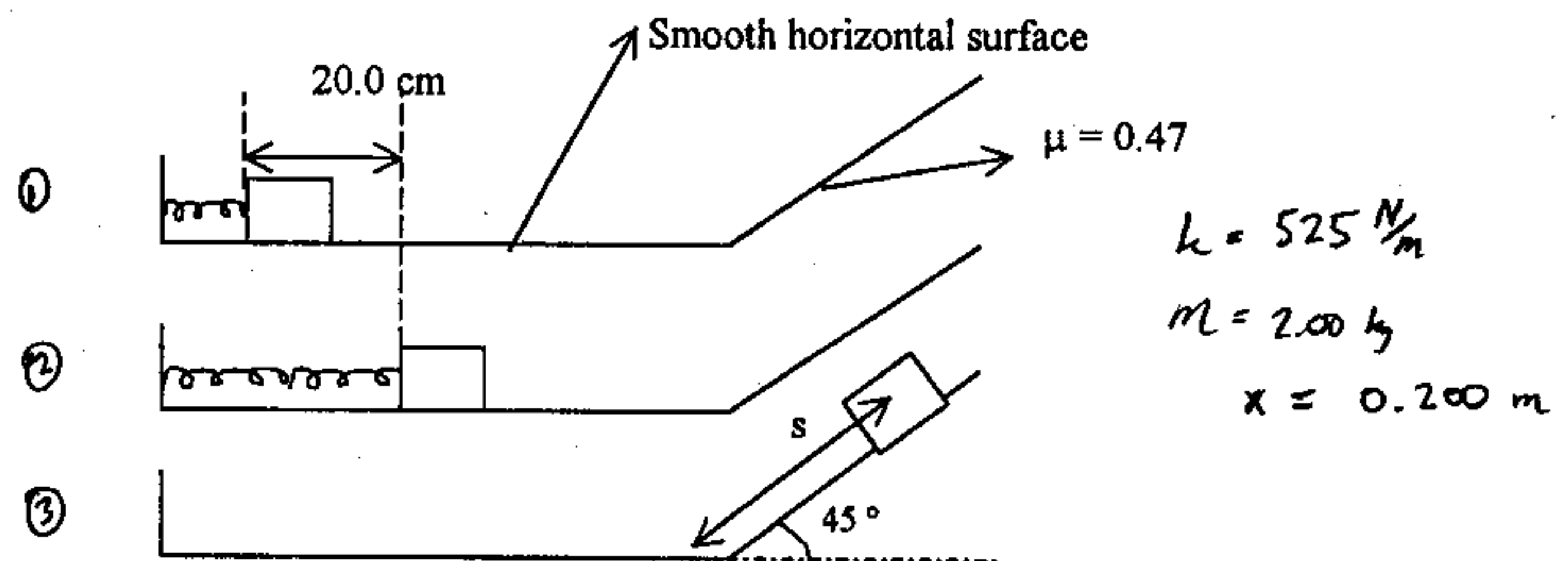
$$T' = 2.69 \times 10^4 \text{ N}$$

and with $v' = 5.00 \text{ m/s}$, power delivered is

$$P' = Fv' = (2.69 \times 10^4 \text{ N})(5.00 \text{ m/s}) = 1.34 \times 10^5 \text{ W}$$



3. A 2.00 kg- block on a frictionless horizontal surface is pushed against a spring that has a spring force constant of 525 N/m, compressing it by 20.0 cm. The block is then released, and the spring projects it along a smooth horizontal surface and up a rough surfaced incline with an angle of 45° as shown below. The coefficient of friction on the incline $\mu = 0.47$.



- a) What is the kinetic energy of the block when the block leaves the spring? (5 points)

Since horiz surface is smooth, energy is conserved between pictures ① and ②
Then $E_1 = E_2$ gives:

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 = K, \text{ So } K = \frac{1}{2}kx^2 = \frac{1}{2}(525 \frac{\text{N}}{\text{m}})(0.200 \text{ m})^2 = \boxed{10.5 \text{ J}}$$

- b) Find the speed of the block when it leaves the spring. (2 points)

Using the result of (a),

$$\frac{1}{2}mv^2 = (10.5 \text{ J}) \Rightarrow v^2 = \frac{2(10.5 \text{ J})}{(2.00 \text{ kg})} = 10.5 \frac{\text{m}^2}{\text{s}^2} \Rightarrow \boxed{v = 3.24 \frac{\text{m}}{\text{s}}}$$

- c) How far up the incline does the block travel? (distance s in the figure) (9 points)

Block goes up incline a distance d , then the increase in height is $d \sin \theta$ and the work done by friction is

$$W_{\text{fric}} = -f_k d$$

$$= -\mu_k mg (\cos \theta) d$$

Also $K=0$ when block is at top of incline.

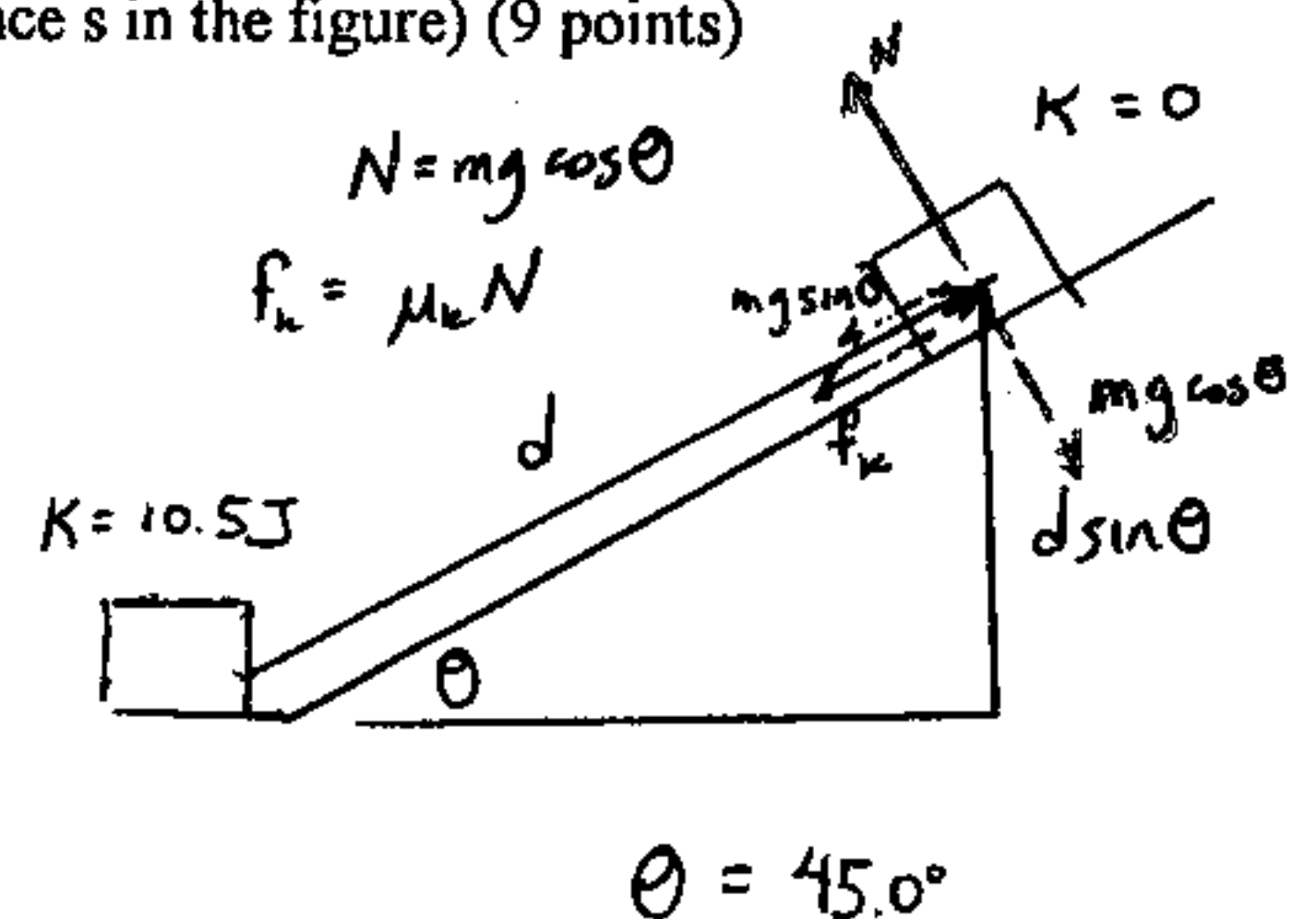
Now set up $\Delta E = \Delta K + \Delta U = W_{\text{fric}}$

$$-10.5 \text{ J} + mg d \sin \theta = -\mu_k mg (\cos \theta) d$$

$$d (mg \sin \theta + \mu_k mg \cos \theta) = 10.5 \text{ J}$$

$$d = \frac{(10.5 \text{ J})}{mg (\sin \theta + \mu_k \cos \theta)} = \frac{(10.5 \text{ J})}{(2.00 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2})(\sin 45^\circ + 0.47 \cos 45^\circ)}$$

$$= \boxed{0.515 \text{ m}}$$



Solve for d:

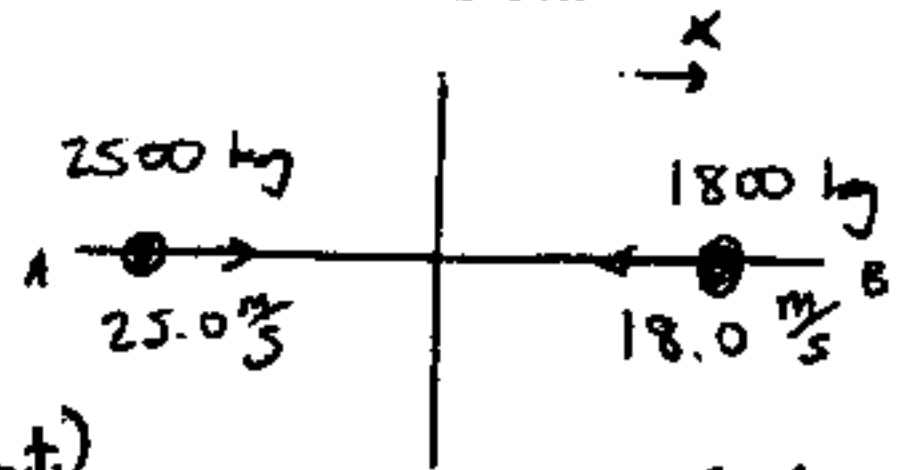
4. Pickup truck A ($M_A=2500$ kg, including driver) is traveling in an easterly (+x) direction at a speed of 25.0 m/s along a country road. Pickup truck B ($M_B=1800$ kg, including driver) is traveling in a westerly (-x) direction with a speed of 18.0 m/s. Since both have their left wheels across the centerline of the road, they collide. Truck A is deflected by 25.0° from its original direction of motion and has a speed of 15.0 m/s immediately after the collision.

- (a) Calculate the velocity of the center of mass of the two-truck system before the collision. (5 points)

Use $\vec{v}_{cm} = \frac{\vec{P}}{M}$ or $\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{M}$:

$$v_{cm,x} = \frac{(2500 \text{ kg})(25.0 \frac{m}{s}) + 1800 \text{ kg}(-18.0 \frac{m}{s})}{4300 \text{ kg}} = 7.00 \frac{m}{s}$$

(There is only an x component.)



- (b) Calculate the velocity of truck B immediately after the collision. (10 points)

Total momentum of the system is conserved. (Isolated system during collision.)

$$\begin{aligned} x: (2500 \text{ kg})(25.0 \frac{m}{s}) + (1800 \text{ kg})(-18.0 \frac{m}{s}) \\ = (2500 \text{ kg})(15.0 \frac{m}{s} \cos 25^\circ) + (1800 \text{ kg})v_x \end{aligned}$$

Solve for v_x . Get:

$$v_x = -2.16 \frac{m}{s}$$

$$y: 0 = (2500 \text{ kg})(15.0 \frac{m}{s} \sin 25^\circ) + (1800 \text{ kg})v_y$$

Solve for v_y . Get

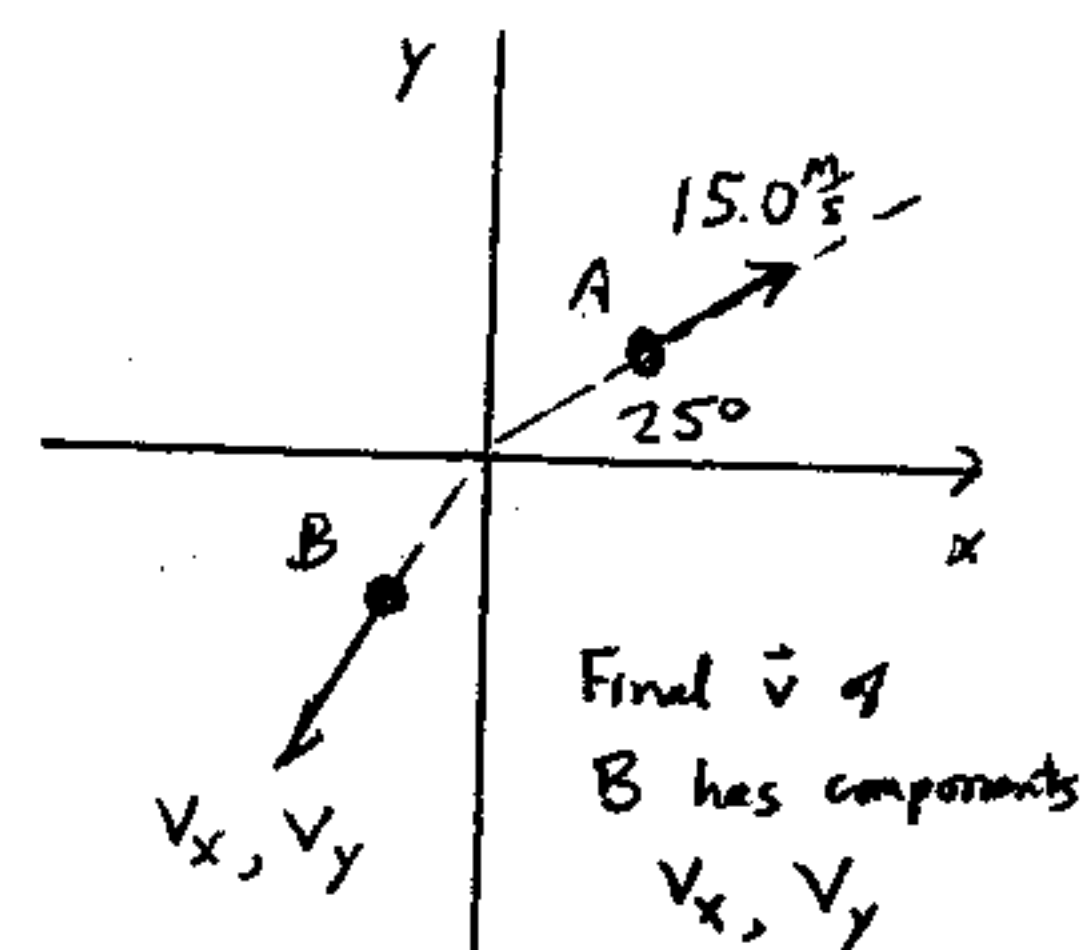
$$v_y = -8.80 \frac{m}{s}$$

Then final velocity of B has magnitude

$$v = \sqrt{v_x^2 + v_y^2} = 9.1 \frac{m}{s}$$

And direction

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) + 180^\circ = 256^\circ = -104^\circ \text{ from } +x \text{ axis}$$



- (c) Determine the velocity of the center of mass of the two-truck system after the collision. You must explain your answer! (5 points)

v_{cm} is the same as given in part (a) ($7.00 \frac{m}{s} \hat{i}$) because $v_{cm} = \frac{\vec{P}}{M}$ and total momentum \vec{P} is the same before & after the collision (as is total mass M !). [This is because external forces can be neglected during collision.]

- (d) The driver of truck A has a mass of 85.0 kg. If the duration of the collision is 0.120 s, what is the magnitude of the average force on this driver during the collision? (8 points)

Since $\vec{F}_{av} = \frac{\Delta \vec{P}}{\Delta t}$, find the change in momentum of driver A:

$$\begin{aligned} \Delta \vec{P}_A &= m_A \Delta \vec{v}_A = (85.0 \text{ kg}) \left((15.0 \frac{m}{s} \cos 25^\circ - 25.0 \frac{m}{s}) \hat{i} + (15.0 \frac{m}{s} \sin 25^\circ) \hat{j} \right) \\ &= \left[-9.69 \times 10^2 \hat{i} + 5.39 \times 10^2 \hat{j} \right] \text{ kg} \frac{m}{s} \end{aligned}$$

Then

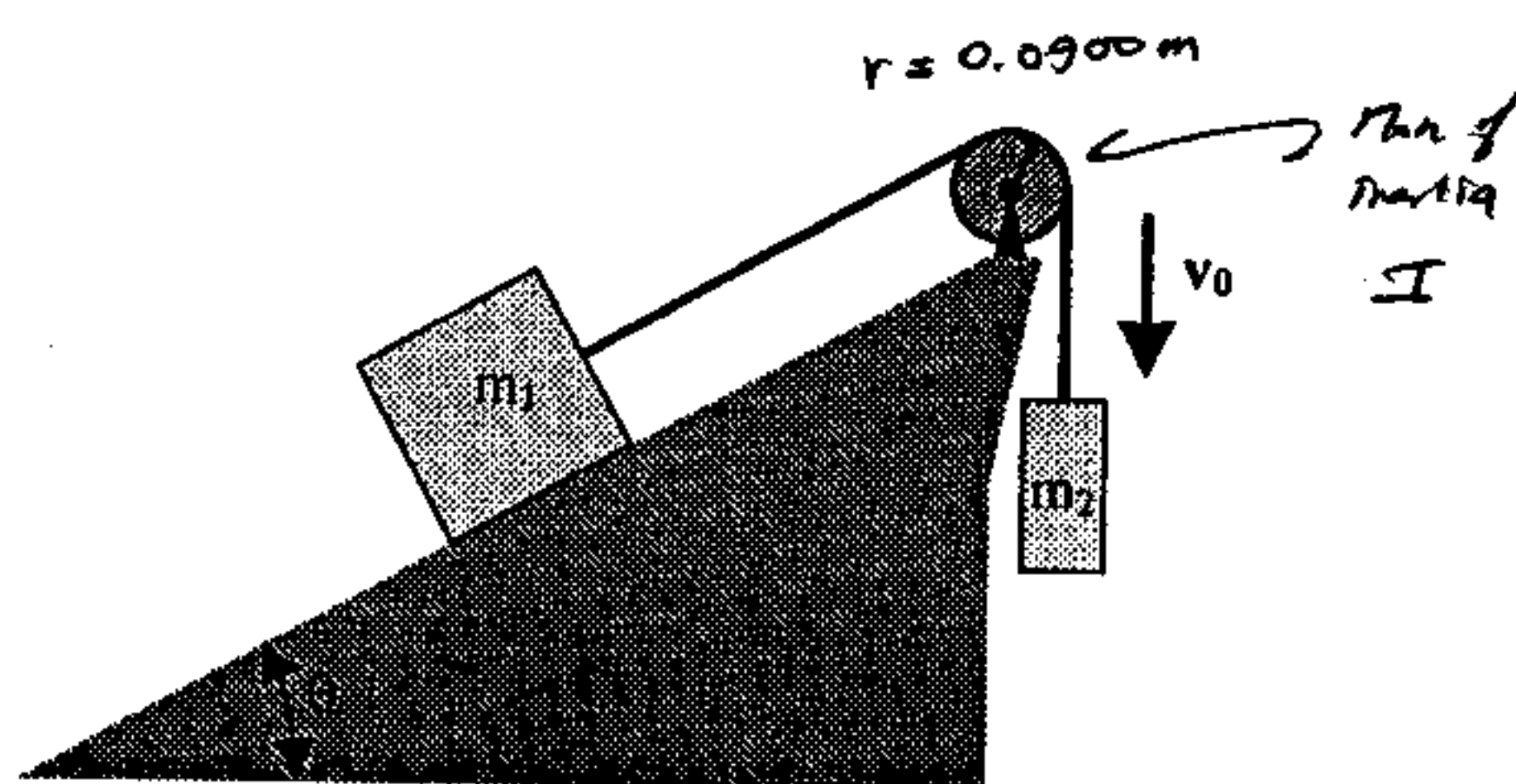
$$\vec{F}_{av} = \frac{\Delta \vec{P}_A}{(0.120 \text{ s})} = \left[-8.03 \times 10^3 \hat{i} + 4.49 \times 10^3 \hat{j} \right] \text{ N}$$

which has magnitude $F_A = 9.24 \times 10^3 \text{ N}$ and dir. 151°

(e) Is this collision elastic? You must justify your answer. (5 points)

We need to compare the final total KE with the initial total KE. If they are different then the collision was inelastic. In fact no calculation is necessary since the speeds of both cars have decreased in the collision. So the total KE has decreased and the collision is inelastic.

5. A light, inextensible string passes over a frictionless pulley and connects masses m_1 and m_2 as shown. The incline is frictionless and makes an angle of $\theta = 30.0^\circ$ with the horizontal. The pulley has a radius of 9.00 cm and a moment of inertia of 0.0450 kg m^2 . At the instant shown, the system is moving in the direction shown (for m_2) with a speed of $v_0 = 0.800 \text{ m/s}$. Given $m_1 = 10.0 \text{ kg}$ and $m_2 = 3.00 \text{ kg}$, how far will m_2 descend from the position shown before it stops? (7 points)



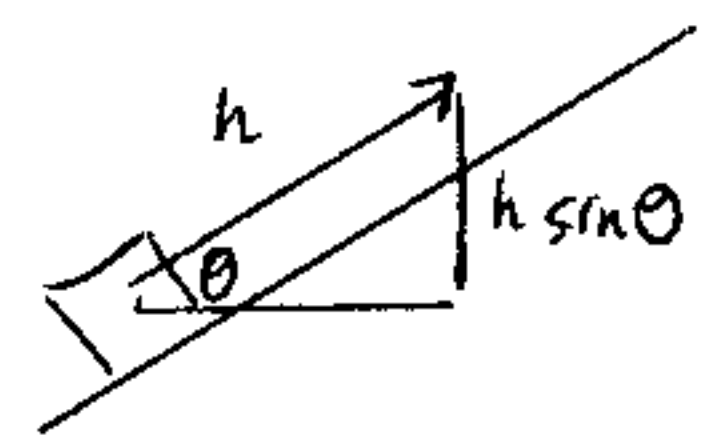
Initially the masses and the pulley are in motion; in the "final" configuration the masses have different heights and all three objects are motionless.

Initial kinetic energy: The masses have the same speed $v = 0.800 \text{ m/s}$ and the angular velocity of the pulley is $\omega = v/r = \frac{0.800 \text{ m/s}}{0.0900 \text{ m}} = 8.88 \text{ /s}$

$$K_{i, \text{Tot}} = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} I \frac{v^2}{r^2}$$

$$= \frac{1}{2} v^2 (m_1 + m_2 + \frac{I}{r^2})$$

Now suppose m_2 drops by a distance h . Then m_2 will go up the slope a distance h and its height increases by $h \sin \theta$.



Then the change in grav. potential energy of the system is

$$\Delta U_{\text{grav}} = \underset{\substack{\uparrow \\ m_2 \text{ drops}}}{-m_2 g h} + m_1 g h \sin \theta = (-m_2 + m_1 \sin \theta) g h$$

As there are no friction forces the total change in mech. energy of the system is zero!

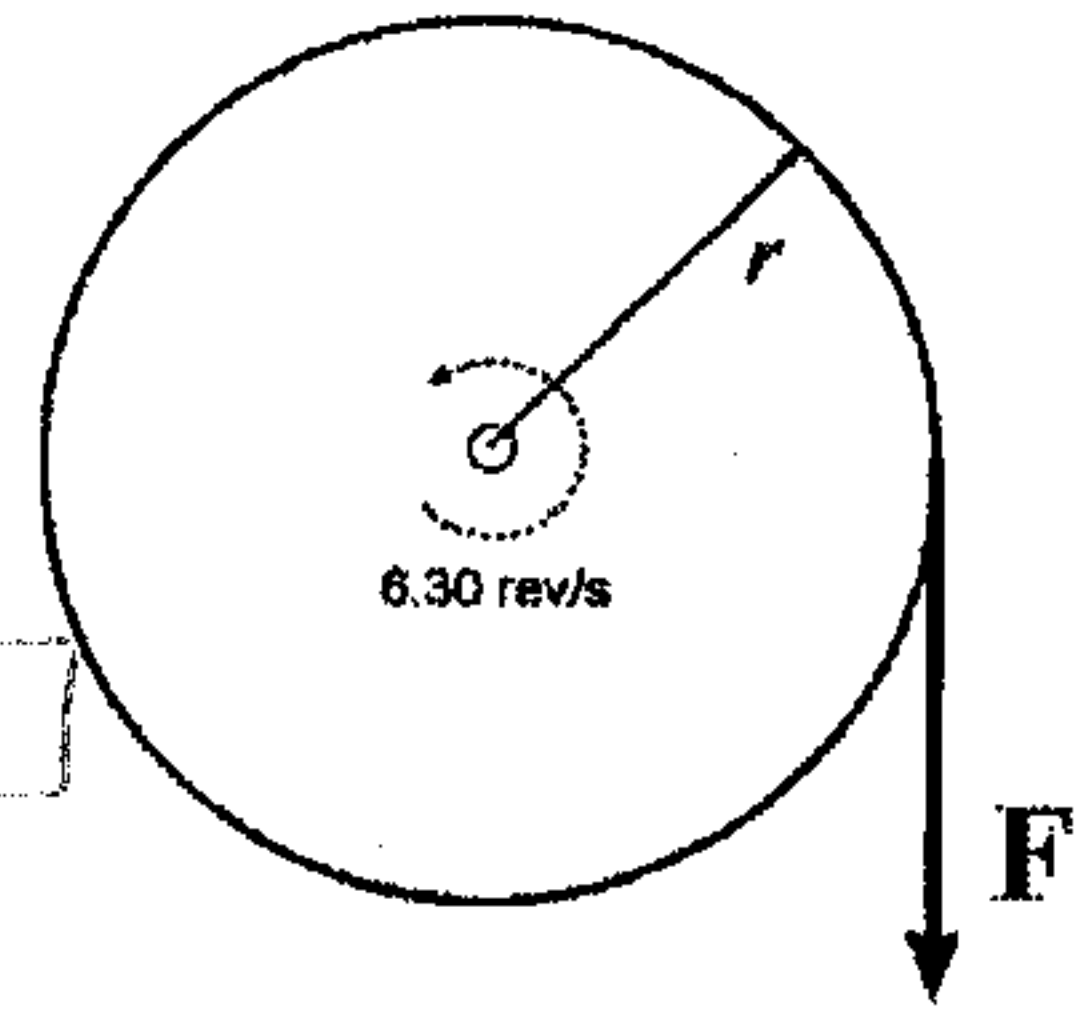
$$\Delta K + \Delta U = 0 - \frac{1}{2} v^2 (m_1 + m_2 + \frac{I}{r^2}) + (-m_2 + m_1 \sin \theta) g h = 0$$

Solve for h :

$$h = \frac{\frac{1}{2} v^2 (m_1 + m_2 + \frac{I}{r^2})}{g (-m_2 + m_1 \sin \theta)} = \frac{\frac{1}{2} (0.800 \text{ m/s})^2 (10.0 \text{ kg} + 3.00 \text{ kg} + \frac{0.0450 \text{ kg m}^2}{(0.0900 \text{ m})^2})}{(9.80 \text{ m/s}^2) (-3.00 \text{ kg} + (10.0 \text{ kg}) \sin 30.0^\circ)}$$

$$= \boxed{0.303 \text{ m}}$$

6. A uniform cylindrical disk of radius 0.150 m and mass 16.0 kg turns about a frictionless axle through its center at a rate of 6.30 $\frac{\text{rev}}{\text{s}}$. By applying a constant tangential retarding force at its edge, it is slowed to a halt in 8.0 s.



- a) Find the moment of inertia of the disk. (2 points)

Disk is a uniform cylinder

$$\Rightarrow I = \frac{1}{2} MR^2 = \frac{1}{2} (16.0 \text{ kg})(0.150 \text{ m})^2 = 0.180 \text{ kg}\cdot\text{m}^2$$

- b) Find the magnitude of the torque acting on the disk during its deceleration. (6 points)

The initial angular velocity is

$$\omega_0 = (6.30 \frac{\text{rev}}{\text{s}}) (\frac{2\pi \text{ rad}}{1 \text{ rev}}) = 39.6 \frac{\text{rad}}{\text{s}} \quad \text{and the final ang vel is } \omega = 0,$$

so α is:

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{-39.6 \frac{\text{rad}}{\text{s}}}{8.0 \text{ s}} = -4.9 \frac{\text{rad}}{\text{s}^2}, \quad \text{and:}$$

$$\tau = I\alpha = (0.180 \text{ kg}\cdot\text{m}^2)(-4.9 \frac{\text{rad}}{\text{s}^2}) = -0.89 \text{ N}\cdot\text{m} \quad |\tau| = 0.89 \text{ N}\cdot\text{m}$$

- c) Find the magnitude of the tangential force which gives this torque. (3 points)

The force pulls tangentially so that $|F_{\perp}| = |F \sin \phi| = F$

$$|\tau| = |Fr \sin \phi| = Fr = 0.89 \text{ N}\cdot\text{m}$$

$$F = \frac{(0.89 \text{ N}\cdot\text{m})}{(0.150 \text{ m})} = 5.9 \text{ N}$$

- d) Find the number of revolutions made by the disk as it slowed to a halt. (3 points)

Units of 'revs' are OK in the kinematic equations, so

$$\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t = 0 + \frac{1}{2}(6.30 \frac{\text{rev}}{\text{s}} + 0)(8.0 \text{ s})$$

$$= 25 \text{ revs}$$

7. A thin rod of length 1.00 m and negligible mass is pivoted about a frictionless joint. At a point 0.750 m from the pivot it supports a 1.50 kg mass, and it is supported at its far end by a cable which makes an angle of 45.0° with the horizontal.

Find the tension in the cable. (6 points)

String attached to the hanging mass must have a tension of

$$mg = (1.50 \text{ kg})(9.80 \frac{\text{m}}{\text{s}^2}) = 14.7 \text{ N}$$

Then the net ccw torque on the rod is

$$-(14.7 \text{ N})(0.750 \text{ m}) + T(1.00 \text{ m})(\sin 45^\circ) = 0$$

(Zero, since rod is static; $\alpha = 0$) Solve for T:

$$T = \frac{(14.7 \text{ N})(0.750 \text{ m})}{(1.00 \text{ m}) \sin 45^\circ} = 15.6 \text{ N}$$

