Worked Examples from Introductory Physics (Algebra–Based) Vol. IV: Electricity

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Contents

Chapter 1

Electric Charge and Coulomb's Law

1.1 The Important Stuff

1.1.1 Electric Charge

In the latter part of the 18th century it was realized that any sample of matter has a property which is as fundamental as its mass. This property is the **electric charge** of the sample. Electric charge can be detected because it gives rise to electric forces. The reason that we don't see electric phenomena more often than we do is that electric charges come in two types —positive and negative— and usually the two types occur in equal numbers so that they add to give zero net charge. But when we can separate positive and negative charges we observe electric forces on a large scale.

In the SI system, electric charge is measured in **Coulombs**. Throughout our study of electromagnetism we will derive other electrical units based on the Coulomb and the units already encountered in mechanics.

After decades of study of the electrical properties of matter, it was found that the fundamental charges in nature occur in integer multiples of the **elementary charge** e,

$$e = 1.602 \times 10^{-19}\,\mathrm{C} \tag{1.1}$$

In discussing this property of charge we often say that electric charge is **quantized**.

In the atom, the nucleus has a charge which is a multiple of $+e$ while the orbiting electrons each have a charge of $-e$. The charge of the nucleus comes from the constituent protons, each of which has a charge of $+e$; the neutrons in the nucleus have no charge.

1.1.2 Some Facts About Electric Charge

Electric charges can be separated by rubbing, as when you rub a plastic rod with some roadkill; see Fig. 1.1. Then one of the objects will obtain a positive charge and the other

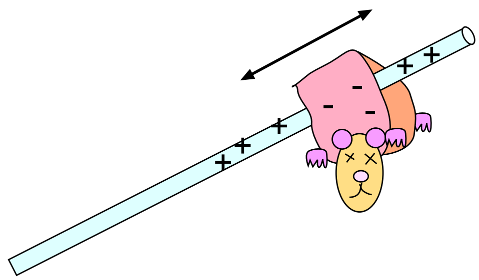

Figure 1.1: Roadkill: Good for separating charges and mighty good eatin'.

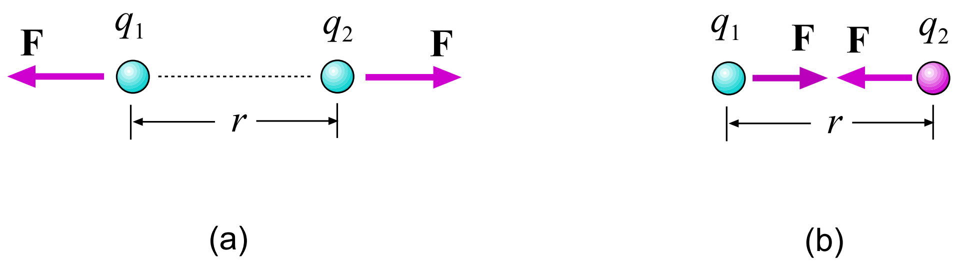

Figure 1.2: (a) Charges q_1 and q_2 have the same sign; the mutual force is repulsive. (b) Charges q_1 and q_2 have opposite signs; the mutual force is attractive.

a negative charge. This occurs because the negatively–charged electrons are *removed* from one object and deposited on the other.

It has been found that in an isolated system the *total* amount of charge stays the same, i.e. total electric charge is *conserved*.

It is also found that electric charges of the same sign (i.e. both positive or both negative) will *repel* and electric charges of opposite sign (i.e. one positive and one negative) will *attract*.

In understanding the behavior of charged objects it is important to understand how charges can move through them. To this end we distinguish objects as being either **conductors** or **insulators**. Excess charge can move freely through a conductor and since like charges repel one another, the charges on a charged conductor will generally move around to space themselves out as much as possible.

In contrast, for insulators excess charge cannot move freely and generally will stay where it is placed.

1.1.3 Coulomb's Law

The force between two small (point) charges is directed along the line which joins the two charges and is repulsive for two charges of the same sign, attractive for two charges of the opposite sign. (See Fig. 1.2. It is proportional to the size of either one of the two charges; finally, it gets weaker as the distance between the charges increases. But the force is not inversely proportional to the distance, it is inversely proportional to the *square* of the distance.

The law for the magnitude of the electric force between two small charges q_1 and q_2 separated by a distance r is

$$F = k\frac{|q_1 q_2|}{r^2} \qquad \text{where} \qquad k = 8.99 \times 10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}} \tag{1.2}$$

This is usually called **Coulomb's law**.

The constant k will come up often in our examples but later on it will be easier to work with the constant ϵ_0, which is related to k by

$$k = \frac{1}{4\pi\epsilon_0}$$

so that ϵ_0 has the value

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ \frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}} \tag{1.3}$$

The electric force given by Coulomb's law is similar to Newton's law for the gravitational force (from first semeseter) in that both are **inverse–square** laws; the force is inversely proportional to the *square* of the distance between the particles.

If we plug some easy numbers into Eq. 1.2 we find that if two 1.0 C charges are separated by a meter, then each one experiences a repulsive force of about 9.0×10^9 N, which is an *enormous* force. In this sense, 1 C is a *huge* amount of charge; typically the charges which one would encounter in real life are of the order of μC (10^{-6} C) or nC (10^{-9} C).

When a charge Q is in the vicinity of several other charges (q_1, q_2, etc.) the net force on Q is found by adding up the individual forces from the other charges. Of course, this is a *vector* sum of the forces.

1.2 Worked Examples

1.2.1 Electric Charge

1. How many electrons must you have to get a total charge of -1.0 C? How many moles of electrons is this?

Since each electron has a charge of -1.6×10^{-19} C, the number of electrons required is

$$N = \frac{(-1.0\,\mathrm{C})}{(-1.6 \times 10^{-19}\,\mathrm{C})} = 6.2 \times 10^{18}$$

A *mole* of any kind of particle is $N_{\mathrm{Avo}} = 6.02 \times 10^{23}$ (Avogadro's number) of those particles. Here we have 6.2×10^{18} electrons and that is

$$n = \frac{N}{N_{\mathrm{Avo}}} = \frac{(6.2 \times 10^{18})}{(6.02 \times 10^{23})} = 1.04 \times 10^{-5}\,\mathrm{moles}$$

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2. A metal sphere has a charge of $+8.0\,\mu\mathrm{C}$. What is the net charge after 6.0×10^{13} electrons have been placed on it? [CJ6 15-2]

The total charge of 6.0×10^{13} electrons is

$$Q_{\mathrm{elec}} = (6.0 \times 10^{13})(-e) = (6.0 \times 10^{13})(-1.60 \times 10^{-19}\,\mathrm{C}) = -9.6 \times 10^{-6}\,\mathrm{C} = -9.6\,\mu\mathrm{C}$$

After this charge has been added to the metal sphere its total charge is

$$Q_{\mathrm{sph}} = +8.0\,\mu\mathrm{C} - 9.6\,\mu\mathrm{C} = -1.6\,\mu\mathrm{C}$$

1.2.2 Coulomb's Law

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3. A charge of 4.5×10^{-9} C is located 3.2 m from a charge of -2.8×10^{-9} C. Find the electrostatic force exerted by one charge on another. [SF7 15-1]

This will be a force of *attraction* between the two charges since they are of opposite signs. The magnitude of this force is given by Coulomb's law, Eq. 1.2,

$$\begin{aligned} F &= k\frac{|q_1 q_2|}{r^2} \\ &= (8.99 \times 10^9\,\tfrac{\mathrm{N \cdot m^2}}{\mathrm{C^2}})\frac{(4.5 \times 10^{-9}\,\mathrm{C})(2.8 \times 10^{-9}\,\mathrm{C})}{(3.2\,\mathrm{m})^2} = 1.1 \times 10^{-8}\,\mathrm{N} \end{aligned}$$

The charges will attract one another with a force of magnitude 1.1×10^{-8} N.

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4. An alpha particle (charge=$+2.0e$) is sent at high speed toward a gold nucleus (charge=$+79e$). What is the electrical force acting on the alpha particle when it is 2.0×10^{-14} m from the gold nucleus? [SF7 15-3]

Here both particles are *positively* charged so there is a force of *repulsion* between them. The magnitude of this force of repulsion is given by Coulomb's law,

$$\begin{aligned} F &= k\frac{|q_1 q_2|}{r^2} = k\frac{(2.0)e(79.0)e}{r^2} \\ &= (8.99 \times 10^9 \, \tfrac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}) \frac{(2.0)(79.0)(1.6 \times 10^{-19}\,\mathrm{C})^2}{(2.0 \times 10^{-14}\,\mathrm{m})^2} = 91.1\,\mathrm{N} \end{aligned}$$

So the alpha particle experiences a (repulsive) force of 91 N from the gold nucleus.

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5. Two identical conducting spheres are placed with their centers $0.30\,\mathrm{m}$ apart. One is given a charge of $12 \times 10^{-9}\,\mathrm{C}$, the other a charge of $-18 \times 10^{-9}\,\mathrm{C}$. (a) Find the electrostatic force exerted on one sphere by the other. (b) The spheres are connected by a conducting wire. Find the electrostatic force between the two after equilibrium is reached. [SF7 15-9]

(a) Use Coulomb's law to find the magnitude of the force, which in this case is attractive since the spheres are oppositely charged:

$$F = k\frac{|q_1 q_2|}{r^2} = (8.99 \times 10^9 \, \tfrac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}) \frac{(12 \times 10^{-9}\,\mathrm{C})(18 \times 10^{-9}\,\mathrm{C})}{(0.30\,\mathrm{m})^2} = 2.16 \times 10^{-5}\,\mathrm{N}$$

(b) When the (conducting) spheres are connect by a (thin!) conducting wire, the electric charges are free to move between the spheres. The total charge on both spheres is

$$Q_{\mathrm{Tot}} = 12 \times 10^{-9}\,\mathrm{C} - 18 \times 10^{-9}\,\mathrm{C} = -6.0 \times 10^{-9}\,\mathrm{C}$$

and when this charge is free to move between the spheres it will attain an equilibrium when both spheres have the *same* charge. So after the spheres are connected the charge of each is

$$Q = Q_{\mathrm{Tot}}/2 = -3.0 \times 10^{-9}\,\mathrm{C}$$

This is shown in Fig. 1.3.

With the new charges on the spheres, use Coulomb's law to get the magnitude of the force on each:

$$F = k\frac{|q_1 q_2|}{r^2} = (8.99 \times 10^9 \, \tfrac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}) \frac{(3.0 \times 10^{-9}\,\mathrm{C})(3.0 \times 10^{-9}\,\mathrm{C})}{(0.30\,\mathrm{m})^2} = 8.99 \times 10^{-7}\,\mathrm{N}$$

and now the force is *repulsive* since both spheres are both negatively charged.

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6. Three charges are arranged as shown in Fig. 1.4. Find the magnitude and

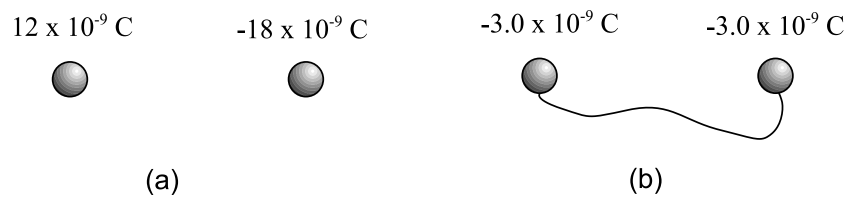

Figure 1.3: (a) Conducting spheres are given different charges. (b) Charges on the spheres after being joined by a conducting wire.

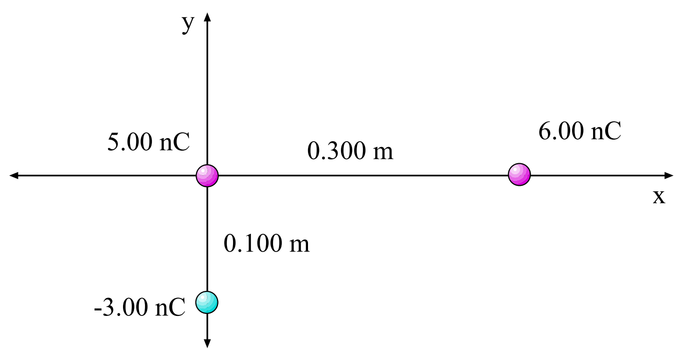

Figure 1.4: Charges in Example 6

direction of the electrostatic force on the charge at the origin. [SF7 15-11]

Let's call the 6.00 nC charge q_1 and the -3.00 nC charge q_2. (The charge at the origin is $Q = +5.00\,\mathrm{nC}$.)

The force from q_1 is *repulsive* and points to the right. The force from q_2 is *attractive* and points downward, as shown in Fig. 1.5. We need to find the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ and then add those two force vectors.

From Coulomb's law we get the magnitude of $\mathbf{F}_1$; since charge q_1 is at a distance $r_1 =$

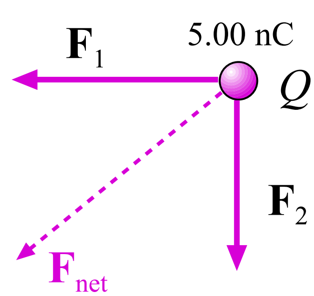

Figure 1.5: Forces on Q in Example 6

0.300 m from Q,

$$F_1 = k\frac{|Qq_1|}{r_1^2} = (8.99 \times 10^9\ \tfrac{\mathrm{N \cdot m^2}}{\mathrm{C^2}})\frac{(5.00 \times 10^{-9}\,\mathrm{C})(6.00 \times 10^{-9}\,\mathrm{C})}{(0.300\,\mathrm{m})^2} = 3.00 \times 10^{-6}\,\mathrm{N}$$

Likewise, the magnitude of $\mathbf{F}_2$ is

$$F_2 = k\frac{|Qq_2|}{r_2^2} = (8.99 \times 10^9\ \tfrac{\mathrm{N \cdot m^2}}{\mathrm{C^2}})\frac{(5.00 \times 10^{-9}\,\mathrm{C})(3.00 \times 10^{-9}\,\mathrm{C})}{(0.100\,\mathrm{m})^2} = 1.35 \times 10^{-5}\,\mathrm{N}$$

Then the *total* force on Q has the components

$$F_x = -3.00 \times 10^{-6}\,\mathrm{N} \qquad F_y = -1.35 \times 10^{-5}\,\mathrm{N}$$

What is the magnitude and direction of this vector? Its magnitude is

$$F = \sqrt{(-3.00 \times 10^{-6}\,\mathrm{N})^2 + (-1.35 \times 10^{-5}\,\mathrm{N})^2} = 1.38 \times 10^{-5}\,\mathrm{N}$$

and the direction we can find from

$$\theta = \tan^{-1}\frac{(-1.35 \times 10^{-5})}{(-3.00 \times 10^{-6})} = 77.5^\circ - 180^\circ = -103^\circ$$

(Note, we subtract 180° from the simple answer because the direction of the force is in the third quadrant.)

The net force on Q has magnitude 1.38×10^{-5} N and points at an angle of -103° from the $+x$ axis.

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7. Two small metallic spheres, each of mass 0.20 g are suspended as pendulums by light strings from a common point as shown in Fig. 1.6. The spheres are given the same electric charge and it is found that they come to equilibrium when each string is at an angle of 5.0° with the vertical. If each string is 30.0 cm long, what is the magnitude of the charge on each sphere? [SF7 15-15]

From simple trig we can calculate the distance between the two spheres. If this distance is x, then

$$x = 2(30.0\,\mathrm{m})\sin 5.0^\circ = 5.23\,\mathrm{cm} = 5.23 \times 10^{-2}\,\mathrm{m}$$

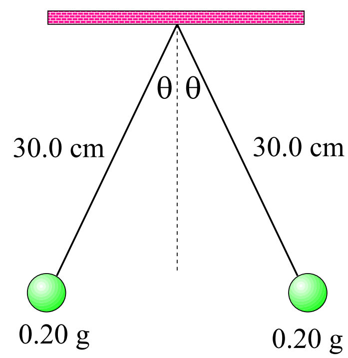

Figure 1.6: Suspended charged spheres in Example 7

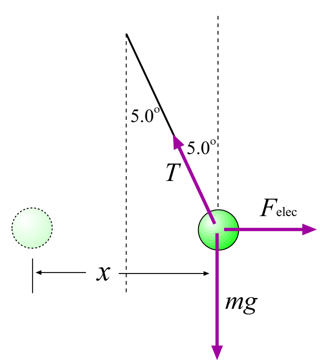

Figure 1.7: Forces acting on a charged sphere in Example 7

Now consider the forces acting on one of the spheres, say the one on the right. These are shown in Fig. 1.7, where we also note (for reference) the location of other sphere. The right sphere experiences a force of electric repulsion from the left sphere. The forces are the force of gravity (mg, downward), the tension of the string (magnitude T; it pulls at an angle 5.0° from the vertical) and the electric repulsive force. From Coulomb's law, the magnitude of the latter is

$$F_{\rm elec} = k\frac{q^2}{x^2}$$

where q is the magnitude of the charge on each sphere.

The sphere is in equilibrium, so the forces must sum to zero. The vertical forces cancel out, giving us:

$$T\cos 5.0^\circ = mg \qquad \Longrightarrow \qquad T = \frac{mg}{\cos 5.0^\circ} = \frac{(2.00\times 10^{-4}\,{\rm kg})(9.80\,\frac{\rm m}{\rm s^2})}{\cos 5.0^\circ} = 1.97\times 10^{-3}\,{\rm N}$$

The horizontal forces cancel out and this gives:

$$T\sin 5.0^\circ = F_{\rm elec} = k\frac{q^2}{x^2}$$

which lets us solve for q:

$$q^2 = \frac{Tx^2\sin 5.0^\circ}{k} = \frac{(1.97\times 10^{-3}\,{\rm N})(5.23\times 10^{-2})^2\sin 5.0^\circ}{(8.99\times 10^9\,\frac{\rm N\cdot m^2}{\rm C^2})} = 5.22\times 10^{-17}\,{\rm C}^2$$

so then

$$q = 7.2\times 10^{-9}\,{\rm C} = 7.2\,{\rm nC}$$

Chapter 2

The Electric Field

2.1 The Important Stuff

2.1.1 The Electric Field

When we solved the longer Coulomb Law problems in the previous chapter we added up the (vector) forces from charges $q_1, q_2, \ldots$ acting on a certain charge Q. Now, each one of these individual forces (and hence the *sum* of those forces) is proportional to the charge Q. If in each of those problems we divided the net force by the charge Q we would get a *force per unit charge* at the location of Q. This quantity (which is a vector, since force is a vector) would depend on the values and locations of the charges $q_1, q_2 \ldots$. This idea is represented in Fig. 2.1.

So, a given configuration of charges $q_1, q_2 \ldots$ gives rise to an **electric field E** such that the force on a charge Q is given by

$$\mathbf{F} = Q\mathbf{E} \quad . \tag{2.1}$$

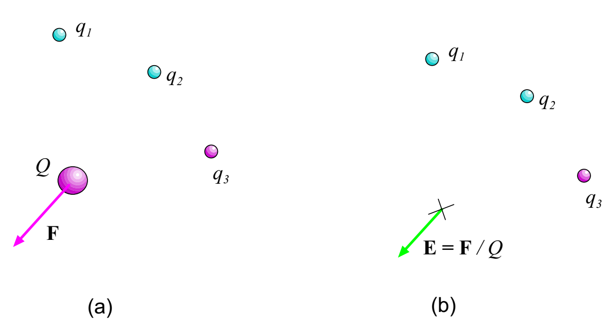

Figure 2.1: (a) Charge Q experiences a force $\mathbf{F}$ from the charges $q_1, q_2 \ldots$. (b) The quantity $\mathbf{E} = \mathbf{F}/Q$ depends only on the charges $q_1, q_2 \ldots$.

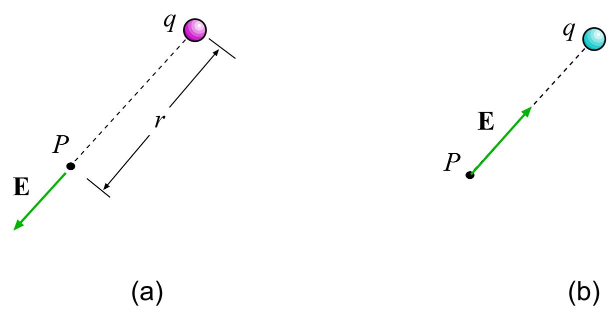

Figure 2.2: (a) Point P is a distance r away from charge q. If q is positive, the electric field points away from q. (b) If q is negative, the electric field points toward q. In both cases the magnitude of $\mathbf{E}$ is given by $E = k|q|/r^2$.

When we use this equation we mean that after we put Q in place all the little charges $q_1, q_2 \ldots$ are in the *same* places they were when we deduced the value of $\mathbf{E}$ from their values and positions! This will be true in practice if the "test charge" Q is small. Thus we give a practical *definition* of the $\mathbf{E}$ field as

$$\mathbf{E} = \frac{\mathbf{F}}{Q} \qquad \text{where } Q \text{ is a } \textit{small} \text{ charge} \tag{2.2}$$

From Eq. 2.2 we see that the electric field is a *vector* and has units of N/C.

We note that finding the electric field is more useful than finding the force on a *specific* charge since once we have the E field we simply multiply by the charge Q to get the force, as given by Eq. 2.1.

2.1.2 Finding the Electric Field

It follows from Coulomb's law that at a point which is a distance r from a point charge q, the magnitude of the electric field is

$$E_{\rm pt-ch} = k\frac{|q|}{r^2} \tag{2.3}$$

and the direction of the field is *away from* q if q is positive and *toward* q if q is negative. This is shown in Fig. 2.2.

When we need to find the electric field due a collection of point charges we find the electric field due to *each* charge and then find the (vector) sum.

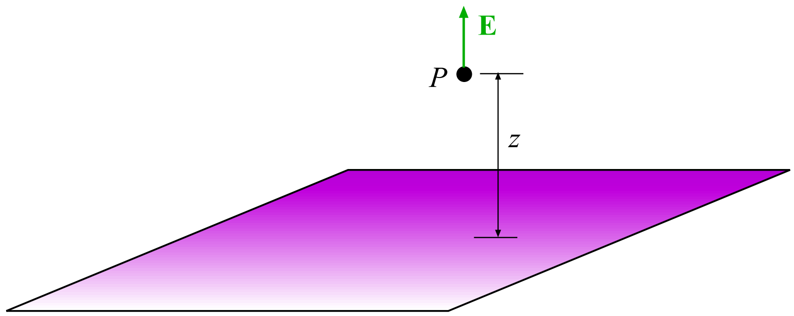

Figure 2.3: Point P is at some distance z above an infinte plane of charge with charge density σ. If σ is positive the E field points away from the sheet and has magnitude $\sigma/(2\epsilon_0)$.

2.1.3 Continuous Distributions; Sheets of Charge

Many charged objects we encounter are not sets of points charges; rather they are continuous distributions of charge. If a two-dimensional region of space contains a charge we can talk about its *charge per unit area*, or **surface charge density**.

Surface charge density is usually given the symbol σ; it has units of $\mathrm{C/m^2}$.

The simplest case of a surface charge is that of an *infinite* planar sheet of charge with uniform charge density σ. We want to know the value of the electric field $\mathbf{E}$ at a point P which is a distance z from the plane; see Fig. 2.3.

It turns out that the answer does not depend on z. If σ is positive, the electric field at P points away from the sheet and has magnitude

$$E_{\mathrm{inf-sh}} = \frac{\sigma}{2\epsilon_0} \quad . \tag{2.4}$$

Here it is easiest to express the result using the constant ϵ_0 introduced in Eq. 1.3

If the sheet has a negative charge density then the field points toward the sheet and the magnitude of the field is $E = |\sigma|/(2\epsilon_0)$.

Next we take the case of the two very large, flat, parallel sheets of charge, as shown in Fig. 2.4. A total charge of $+Q$ has been placed on one sheet and a charge of $-Q$ on the other. We assumed the charge is spread around uniformly so that the charge density of the positively-charge sheet is $\sigma = \frac{Q}{A}$.

This situation can arise when equal and opposite charges are placed on metal plates which are held apart at some distance. (Such a device is called a **parallel–plate capacitor**.) Our approximation is suitable for the case where the plates are separated by a distance which is small compared with the linear size of the plates.

From Eq. 2.4 it follows that the magnitude of the E field between the plates is twice that of the single sheet,

$$E_{\mathrm{inf-sh}} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \quad . \tag{2.5}$$

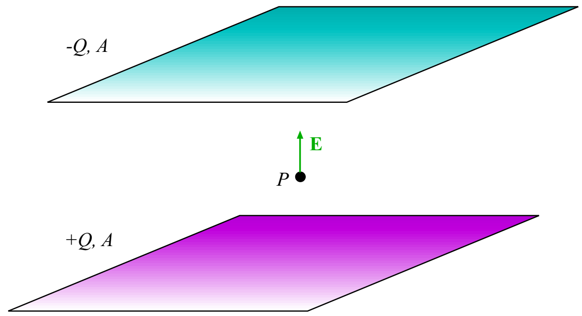

Figure 2.4: Point P is between two very large sheets of charge. On one sheet the total charge is $+Q$ and on the other it is $-Q$. Both sheets have area A. With $\sigma = Q/A$, the electric field between the plates has magnitude σ/ϵ_0.

and the field points from the positive plate to the negative plate.

This equation gives the E field *anywhere* between the plates and it is good as long we can approximate the plates as "very large". Near the edges of the plates it is not a very good approximation.

2.1.4 Electric Field Lines

While the direction of the electric field near a point charge or between two large plates has a simple answer, most charge distributions produce electric fields dependence on position can be hard to visualize.

To help in seeing the direction of the electric at all points we imagine finding the direction of the electric field at all points in space, represented by a little arrow at any point. Then if we join nearby arrows together to form a curve we get an **electric field line**. This is shown in Fig. 2.5 for a (positive) point charge; the field lines start on the charge and go outward. (For a negative charge the field lines would go inward to the charge.)

An interesting and important configuration of charges is the **electric dipole** which consists of two opposite charges $\pm q$ separated by a distance which is usually taken to be "small" in some sense. Near the charge $+q$ the electric field points mainly away from the charge and near the charge $-q$ the field points mainly toward the charge. At other points in space we have to form the sum of the field from the two charges and add. The result is shown in Fig. 2.6.

The mathematics of the electric force gives the following properties of field lines:

• Field lines begin and end *only* on charges; they start on positive charges and end on negative charges.

• Field lines cannot cross one another.

Field lines give us the *direction* of the electric field at any point, but since we have joined the arrows togther to form them, a single field line can't tell the *magnitude* of the E field.

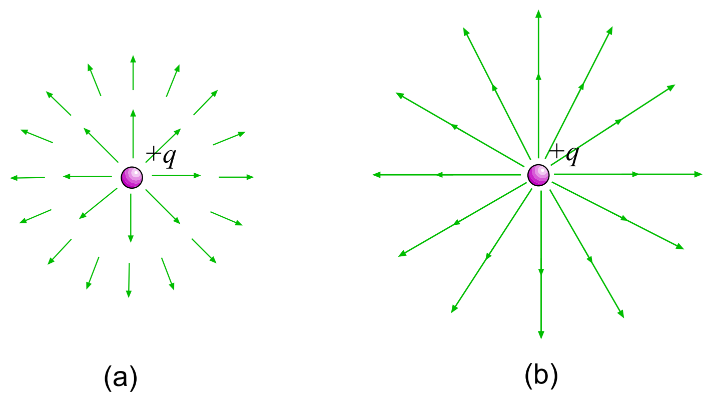

Figure 2.5: (a) A representation of the electric field around a point charge using individual vectors. (b) Representation of the electric field around a point charge using field lines.

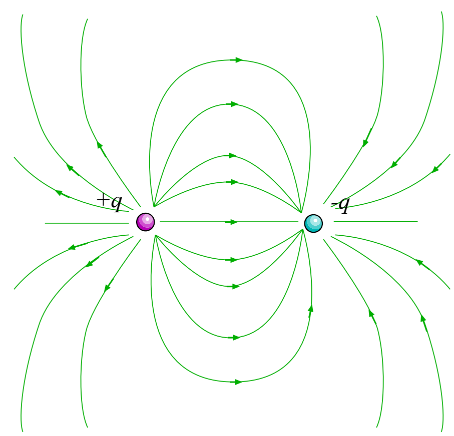

Figure 2.6: Field lines of an electric dipole.

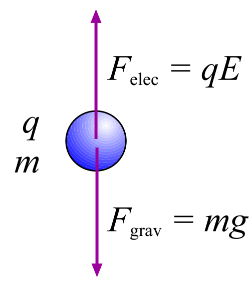

Figure 2.7: Forces on the charged mass in Example 1. The electric force is upward (in the same direction as the E field). The force of gravity is downward.

But the mathematics of the electric force tell us that the number of field lines that we draw originating on a charge should be *proportional* the the size of the charge. If we follow that rule, then the magnitude of the electric field can be judged from the density of field lines at any point. If the lines are closely space, the electric field is strong at that place.

2.1.5 Conductors

In conductors any excess charge is free to move through the material.

2.2 Worked Examples

2.2.1 The Electric Field

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1. An object with a net charge of $24\,\mu\text{C}$ is placed in a uniform electric field of $610\,\frac{\text{N}}{\text{C}}$, directed vertically. What is the mass of the object if it "floats" in the electric field? [SF7 15-17]

The forces acting on this object (of mass m and charge q are shown in Fig. 2.7. The force of gravity has magnitude mg and points downward. The electric force, from Eq. 2.1 has magnitude qE and points upward. (Here the charge q is positive so that the force points in the same direction as the E field.)

The object "floats" so the net force on it must be zero. Hence:

$$qE = mg \qquad \Longrightarrow \qquad m = \frac{qE}{g}$$

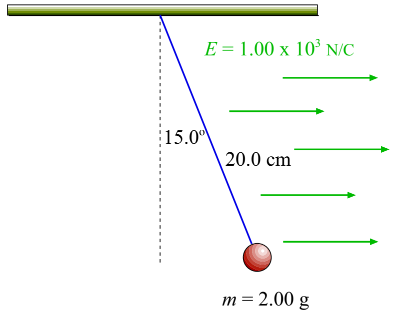

Figure 2.8: Plastic ball suspended in uniform E field, in Example 3.

Plug in the numbers:

$$m = \frac{(24 \times 10^{-6}\,\mathrm{C})(610\,\frac{\mathrm{N}}{\mathrm{C}})}{(9.80\,\frac{\mathrm{m}}{\mathrm{s}^2})} = 1.5 \times 10^{-3}\,\mathrm{kg}$$

So the mass of the object is 1.5 grams.

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2. An electric field of $260000\,\frac{\mathrm{N}}{\mathrm{C}}$ points due west at a certain spot. What are the magnitude and direction of the force that acts on a charge of $-7.0\,\mu\mathrm{C}$ at this spot? [CJ6 18-25]

From $\mathbf{F} = q\mathbf{E}$, the *magnitude* of the force is

$$F = |q|E = (7.0\,\mu\mathrm{C})(260000\,\tfrac{\mathrm{N}}{\mathrm{C}}) = (7.0 \times 10^{-6}\,\mathrm{C})(2.60 \times 10^{5}\,\tfrac{\mathrm{N}}{\mathrm{C}}) = 1.8\,\mathrm{N}$$

Since the charge q is *negative* here, the direction of the force is opposite that of the field $\mathbf{E}$, so the force points to the *East*.

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3. A small 2.00-g plastic ball is suspended by a 20.0-cm -long string in a uniform electric field, as shown in Fig. 2.8. If the ball is in equilibrium when the string makes a 15.0° angle with the vertical as indicated, what is the net charge on the ball? [SF7 15-50]

First, make a free–body diagram of the forces acting on the ball. They are: The string tension T directed along the string; the force of gravity, mg, downward; and the electric force which must be parallel to the electric and so here it must point to the *right*. These forces are shown in Fig. 2.9. The magnitude of the electric force is qE, where q is the charge on the plastic ball; this charge must be positive since the force points in the same direction

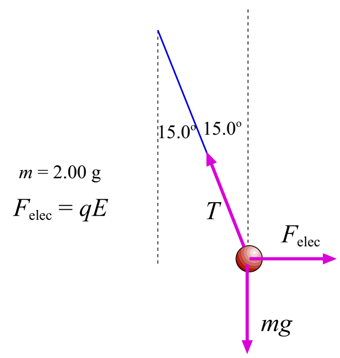

Figure 2.9: Forces acting on the plastic ball in Example 3.

as **E**. The ball is in equilibrium so the (vector) sum of the forces is zero. The condition that the vertical force components sum to zero allows us to find T:

$$T\cos 15.0^\circ - mg = 0 \qquad \Longrightarrow \qquad T = \frac{mg}{\cos 15.0^\circ} = \frac{(2.00\times 10^{-3}\,\mathrm{kg})(9.80\,\frac{\mathrm{m}}{\mathrm{s}^2})}{(\cos 15.0^\circ)} = 2.03\times 10^{-2}\,\mathrm{N}$$

The condition that the horizontal forces sum to zero gives us:

$$-T\sin 15^\circ + F_{\mathrm{elec}} = -T\sin 15^\circ + qE = 0 \qquad \Longrightarrow \qquad q = \frac{T\sin 15^\circ}{E}$$

Plug in the numbers and get:

$$q = \frac{(2.03\times 10^{-2}\,\mathrm{N})\sin 15^\circ}{(1.00\times 10^{-3}\,\frac{\mathrm{N}}{\mathrm{C}})} = 5.25\times 10^{-6}\,\mathrm{C} = 5.25\,\mu\mathrm{C}$$

---

4. Each of the protons in a particle beam has a kinetic energy of 3.25×10^{-15} J. What are the magnitude and direction of the electric field that will stop these protons in a distance of $1.25\,\mathrm{m}$? [SF7 15-22]

First, use the proton mass and definition of kinetic to find the initial speed of these protons. With $m_{\mathrm{p}} = 1.67\times 10^{-27}\,\mathrm{kg}$, we find:

$$\mathrm{KE} = \tfrac{1}{2}m_{\mathrm{p}}v^2 = 3.25\times 10^{-15}\,\mathrm{J} \qquad \Longrightarrow \qquad v^2 = \frac{2(3.25\times 10^{-15}\,\mathrm{J})}{(1.67\times 10^{-27}\,\mathrm{kg})} = 3.89\times 10^{12}\,\frac{\mathrm{m}^2}{\mathrm{s}^2}$$

Then:

$$v = 1.97\times 10^{6}\,\frac{\mathrm{m}}{\mathrm{s}}$$

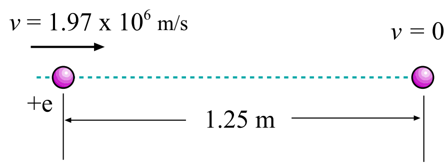

Figure 2.10: Proton slows to a halt in Example 4.

The motion of the proton is as shown in Fig. 2.10. Using our equations of kinematics, we can find the acceleration of the proton:

$$v^2 = v_0^2 + 2ax \qquad \Longrightarrow \qquad a = \frac{v^2 - v_0^2}{2x} = \frac{0 - (1.97 \times 10^6\ \frac{\mathrm{m}}{\mathrm{s}})^2}{2(1.25\,\mathrm{m})} = -1.56 \times 10^{12}\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

which should be negative since the proton's velocity decreases.

The force on the proton comes from the electric field, as given by Eq. 2.1:

$$F_x = ma_x = qE_x = +eE_x$$

where we've used the fact that a proton's charge is $+e$. Then:

$$E_x = \frac{ma_x}{e} = \frac{(1.67 \times 10^{-27}\,\mathrm{kg})(-1.56 \times 10^{12}\ \frac{\mathrm{m}}{\mathrm{s}^2})}{(1.60 \times 10^{-19}\,\mathrm{C})} = -1.62 \times 10^4\ \frac{\mathrm{N}}{\mathrm{C}}$$

The electric field has magnitude $1.62 \times 10^4\ \frac{\mathrm{N}}{\mathrm{C}}$ and points in the $-x$ direction, that is, opposite the initial motion of the proton.

---

5. A proton accelerates from rest in a uniform electric field of $640\ \frac{\mathrm{N}}{\mathrm{C}}$. At some time, its speed is $1.20 \times 10^6\ \frac{\mathrm{m}}{\mathrm{s}}$. (a) Find the magnitude of the acceleration of the proton. (b) How long does it take the proton to reach this speed? (c) How far has it moved in that interval? (d) What is its kinetic energy at the later time? [SF7 15-23]

(a) The facts given in the problem are diagrammed in Fig. 2.11. If the E field points in the $+x$ direction, then from Eq. 2.1 the force on the proton is

$$F_x = qE_x = +eE_x$$

and the acceleration of the proton is

$$a_x = \frac{F_x}{m_\mathrm{p}} = \frac{eE_x}{m_\mathrm{p}}$$

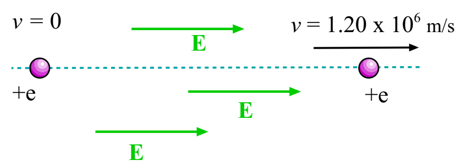

Figure 2.11: Proton is accelerated by E field in Example 5.

Use $m_{\rm p} = 1.67 \times 10^{-27}$ kg and get:

$$a_x = \frac{(1.60 \times 10^{-19}\,\mathrm{C})(640\,\frac{\mathrm{N}}{\mathrm{C}})}{(1.67 \times 10^{-27}\,\mathrm{kg})} = 6.13 \times 10^{10}\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

(b) We have the (constant) acceleration of the proton and its initial and final speeds so using one of our equations from kinematics we can find the distance it traveled:

$$v_x^2 = v_{0x}^2 + 2a_x x \qquad \Longrightarrow \qquad x = \frac{(v_x^2 - v_{0x}^2)}{2a_x}$$

Plug in the numbers:

$$x = \frac{(1.20 \times 10^6\,\frac{\mathrm{m}}{\mathrm{s}})^2 - 0}{2(6.13 \times 10^{10}\,\frac{\mathrm{m}}{\mathrm{s}^2})} = 11.7\,\mathrm{m}$$

(c) Use the definition of kinetic energy, $\mathrm{KE} = \frac{1}{2}mv^2$ and get:

$$\mathrm{KE} = \tfrac{1}{2}(1.67 \times 10^{-27}\,\mathrm{kg})(1.20 \times 10^6)^2 = 1.20 \times 10^{-15}\,\mathrm{J}$$

2.2.2 Finding the Electric Field

---

6. Three point charges are aligned along the x-axis as shown in Fig. 2.12. Find the electric field at the position $x = +2.0\,\mathrm{m}$, $y = 0$. [SF7 15-49]

The point at which we want to calculate the E field, $(2.0\,\mathrm{m}, 0)$, lies to the right of all the charges. At that point, the field due to the $-4.0\,\mathrm{nC}$ charge must point to the *left* since it is a negative charge. That charge lies at a distance of 2.50 m from So x-component of its contribution is

$$E_{1,x} = k\frac{|q_1|}{r_1^2} = (8.99 \times 10^9\,\frac{\mathrm{N \cdot m^2}}{\mathrm{C}^2})\frac{(4.0 \times 10^{-9}\,\mathrm{C})}{(2.50\,\mathrm{m})^2} = -5.75\,\frac{\mathrm{N}}{\mathrm{C}}$$

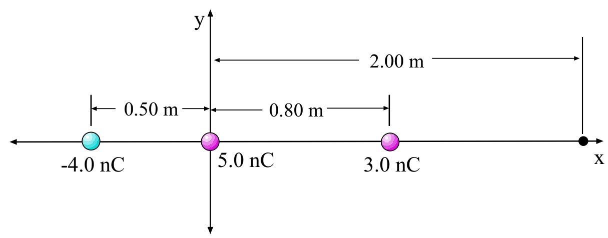

Figure 2.12: Configuration of charges for Example 6.

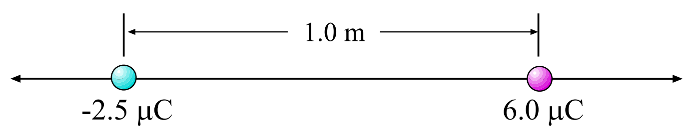

Figure 2.13: Configuration of charges for Example 7.

The field due to the charge at the origin must point to the *right* since it is a positive charge. The x-component of its contribution is

$$E_{2,x} = k\frac{|q_2|}{r_2^2} = (8.99 \times 10^9 \, \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}) \frac{(5.0 \times 10^{-9} \, \mathrm{C})}{(2.00 \, \mathrm{m})^2} = +11.2 \, \frac{\mathrm{N}}{\mathrm{C}}$$

Finally, the field due to the 3.0 nC charge must also point to the *right* since it is a positive charge. This charge's distance from our "observation" point is 1.20 m, so the x-component of its contribution is

$$E_{3,x} = k\frac{|q_3|}{r_3^2} = (8.99 \times 10^9 \, \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}) \frac{(3.0 \times 10^{-9} \, \mathrm{C})}{(1.20 \, \mathrm{m})^2} = +18.7 \, \frac{\mathrm{N}}{\mathrm{C}}$$

Add these up, and the total E field at the given point is

$$E_x = -5.75 \, \frac{\mathrm{N}}{\mathrm{C}} + 11.2 \, \frac{\mathrm{N}}{\mathrm{C}} + 18.7 \, \frac{\mathrm{N}}{\mathrm{C}} = +24.1 \, \frac{\mathrm{N}}{\mathrm{C}}$$

---

7. In Fig. 2.13, determine the point (other than infinity) at which the total electric field is zero. [SF7 15-27]

For all points that we consider there will be a (vector) electric field due to the $-2.5 \, \mu\mathrm{C}$ charge and one due to the $+6.0 \, \mu\mathrm{C}$ charge; we want to find the point at which these vectors add to zero.

It would seem that this point should lie on the line joining the two charges, but do we need to consider points off this line? No, because at points off this axis the two field vectors will point toward or away from the individual charges and at points off the axis those vectors can't be parallel and so can't cancel. So we only need to think about points on the axis.

Could this point lie between the two charges? In that region, the field due to the $-2.5\,\mu\mathrm{C}$ charge will point *toward* that charge (i.e. to the left) and that due to the $+6.0\,\mu\mathrm{C}$ charge will point *away* from that charge (i.e. also to the left). Those vectors can't cancel regardless of their magnitudes, so the point can't lie between the two charges.

How about someplace to the right of both charges? In that region, the $+6.0\,\mu\mathrm{C}$ charge is always closer than the $-2.5\,\mu\mathrm{C}$ charge. That being the case, the field from the $+6.0\,\mu\mathrm{C}$ charge must always have the larger magnitude (charge is bigger and distance is smaller) so again the vectors can't cancel.

The point we want must lie to the left of both charges. In that region, the field due to the $-2.5\,\mu\mathrm{C}$ charge points to the right and that due to the $+6.0\,\mu\mathrm{C}$ charge points to the left. (Note that the $-2.5\,\mu\mathrm{C}$ charge is always *closer* and since it also has a smaller charge, there could be some place where the fields cancel.) If we consider a point which lies at a distance d to the left of the $-2.5\,\mu\mathrm{C}$ charge, then its distance from the $+6.0\,\mu\mathrm{C}$ charge will be $d + 1.0\,\mathrm{m}$, and using Eq. 2.3 the x component of the total field will be

$$E_{x,\mathrm{total}} \;=\; k\frac{(+2.5\,\mu\mathrm{C})}{d^2} - k\frac{(6.0\,\mu\mathrm{C})}{(d+1.0\,\mathrm{m})^2} \;=\; 0 \tag{2.6}$$

It is now just a *math* problem to solve for d. We're done with the *physics*.

First off, we can cancel the constant k in Eq. 2.6 as well as the "μC" units. One trick that will work (unless you've got any better ideas!) is to multiply both sides of Eq. 2.6 by $d^2(d + 1.0\,\mathrm{m})^2$. That gives us:

$$d^2(d+1.0\,\mathrm{m})^2\frac{(+2.5)}{d^2} - d^2(d+1.0\,\mathrm{m})^2\frac{(6.0)}{(d+1.0\,\mathrm{m})^2} \;=\; 0$$

Cancel things and get:

$$(d+1.0\,\mathrm{m})^2(2.5) - d^2(6.0) \;=\; 0$$

which you might recognize as a quadratic equation, so that we *can* get an answer. Expand the square:

$$(2.5)(d^2 + (2.0\,\mathrm{m})d + 1.0\,\mathrm{m}^2) - (6.0)d^2 \;=\; 0$$

and ignoring the "m" units symbol for now, multiply and get:

$$2.5d^2 + 5.0d + 2.5 - 6.0d^2 \;=\; -3.5d^2 + 5.0d + 2.5 \;=\; 0$$

or, without the leading minus sign,

$$3.5d^2 - 5.0d - 2.5 \;=\; 0$$

Almost there! Use the quadratic formula to find:

$$\frac{+5.0 \pm \sqrt{25.0+35}}{7.0} = 1.8\,\mathrm{m}$$

Here we've considered only the "+" root since the other would give a negative value for d which we assumed was *positive*.

So the point we want is 1.8 m to the left of the $-2.5\,\mu\mathrm{C}$ charge.

Chapter 3

Electric Potential Energy; Electric Potential

3.1 The Important Stuff

3.1.1 Electric Potential Energy and Electric Potential

The last two chapters have dealt with *forces* and electric charge. The fundamental equation was Coulomb's law, by way of which we came to talk about the electric field as a quantity of greater utility.

In the first semester of the course, after discussing forces we discovered that *energy* (both potential and kinetic) were useful ideas, and we now discuss the role of energy in electricity.

A charge q moving through an electric field $\mathbf{E}$ experiences a force and so (in general) *work* is done on the charge. An example is shown in Fig. 3.1, where a charge q moves in a

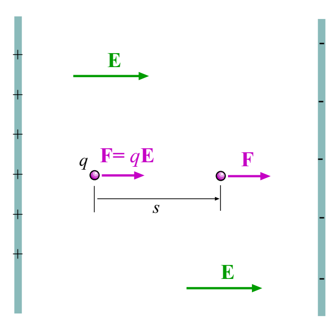

Figure 3.1: A positive charge q moves in a straight line between two large parallel plates, where the E field has a constant value; the force on the charge is also constant and equal to $q\mathbf{E}$. If q moves a distance s as shown, the work done on the charge is $W = Fs = qEs$.

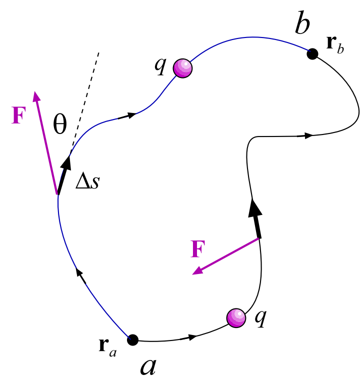

Figure 3.2: Charge q is moved from a to b by two paths. The work done the by electric force on q must be calculated by adding up $\mathbf{F}\Delta s\cos\theta$ for small steps $\Delta\mathbf{s}$ along the paths. Because of a special property of the electric force the value is the same for both paths.

uniform field $\mathbf{E}$. If it moves a distance s in the direction of the field, the work done on the charge is $W = Fs = qEs$.

Generally electric fields are not this simple; they are not uniform and so calculating the work in each case would be hard (it would require calculus!). But we are assured of a couple things:

• The work done on charge q is proportional to q.

• If the other charges producing the electric field stay in their places then the work done by the electric force as q moves from point a to point b *does not depend* on the path taken from a to b.

The second point is illustrated in Fig. 3.2. Here we pick two paths from a to b. The work done by the electric force might be a complicated thing to calculate but we get the same thing in both cases; the value depends only on the endpoints a and b.

Rather than calculate the work done by the electric forces it is easiest to think in term of a **electric potential energy** $\mathrm{PE}_{\mathrm{elec}}$ which can be evaluated at all points in space. The relation between the two is

$$W_{a\to b} = -(\mathrm{PE}_{\mathrm{elec}}(b) - \mathrm{PE}_{\mathrm{elec}}(a)) == -\Delta\mathrm{PE}_{\mathrm{elec}} \tag{3.1}$$

Of course, the electric potential energy $\mathrm{PE}_{\mathrm{elec}}$ is a scalar with units of joules.

Now we deal with the first of the two points: As charge q moves from a to b the work done (and hence the change in potential energy) is proportional to q. If we were to divide $\Delta\mathrm{PE}_{\mathrm{elec}}$ by q we would get a number which does not depend on q, just all of the *other* charges in the world and points a and b. If we call this quantity ΔV, then

$$\Delta V = \frac{\Delta\mathrm{PE}_{\mathrm{elec}}}{q} \tag{3.2}$$

The quantity V is called the **electric potential**, which should *not* be confused with *electric potential energy*. The two are related, but the electric potential V give the *potential energy per unit charge*, just as the electric field $\mathbf{E}$ gave the electric force *per units charge*.

The definition given in Eq. 3.2 only gives the *difference* in values of V, just as it is only the *differences* in the potential energy $\mathrm{PE_{elec}}$ that have any real meaning.

V is a scalar and from Eq. 3.2 it must have units of J/C. Because V is such an important quantity, we give this combination of units a special name, the volt:

$$1\,\mathrm{volt} = 1\,\mathrm{V} = 1\,\tfrac{\mathrm{J}}{\mathrm{C}} \tag{3.3}$$

It will often happen that we will discuss the change in potential energy of an elementary charge (like an electron or a proton) when it moves through a potential difference of 1 volt. When this happens the change in *potantial energy* has magnitude

$$\Delta\mathrm{PE_{elec}} = |q\Delta V| = (e)(1\,\mathrm{V}) = (1.602\times10^{-19}\,\mathrm{C})(1\,\mathrm{V}) = 1.602\times10^{-19}\,\mathrm{J}$$

This gives us a convenient unit of energy for this process; since the energy here is "one electron times one volt" we define a unit of energy called the **electron volt**:

$$1\,\mathrm{electron\ volt} = 1\,\mathrm{eV} = 1.602\times10^{-19}\,\mathrm{J} \tag{3.4}$$

For example when an electron (charge $-e$) goes through a potential gain of $+5.0\,\mathrm{V}$ it has a change in potential energy of $-5.0\,\mathrm{eV}$.

3.1.2 Calculating the Electric Potential

In certain simple situations we can calculate the electric potential

If the point P is a distance r from a point charge q, then the potential at P is given by

$$V = k\frac{q}{r} \tag{3.5}$$

Keep in mind that V is a *scalar* (a single number; no direction), and even though Eq. 3.5 looks like the formula for the *electric field* near a point charge (it should; they are related), we are calculating a it different quantity here. Note, there is only a single power of r in the denominator.

To get the electric potential for a point P which is in the vicinity of a *group* of point charges $q_1, q_2, \ldots$, just add up the electric potentials due to each charge:

$$V = k\frac{q_1}{r_1} + k\frac{q_2}{r_2} + \cdots \tag{3.6}$$

where r_1 is the distance from P to q_1, r_2 is the distance from P to q_2, and so on.

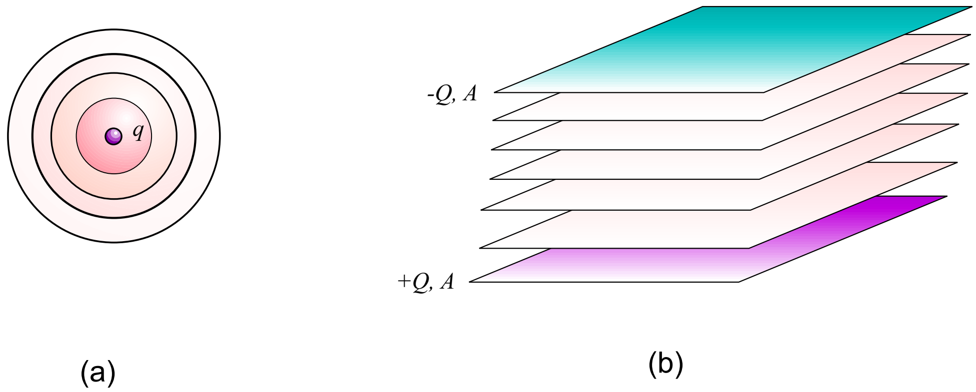

Figure 3.3: Equipotential surfaces for two simple charge configurations. (a) Around a point charge the equipotentials are spherical surfaces. (b) Between two parallel charged plates the equiotentials are parallel planes.

Between two large oppositely-charged parallel plates the electric field is uniform; if the coordinate axis perpendicular to the plates is z and $\mathbf{E}$ points in the positive z direction then the work done by the electric field as a charge q has a change in position given by Δz is

$$W = (qE)\Delta z \qquad \Longrightarrow \qquad \Delta \mathrm{PE}_{\mathrm{elec}} = -W = -qE\Delta z = \Delta(-qEz)$$

This means that we can take the electric potential between the plates as

$$V_{\mathrm{plates}} = -Ez \tag{3.7}$$

Here, the z axis points from the positive plate to the negative plate. If we are given the potential difference of the plates themselves and the distance between the plates ($\Delta z = d$) then we can find the magnitude of $\mathbf{E}$ from: $E = \Delta V/d$.

3.1.3 Equipotentials; Relation Between E and V

For any configuration of charges we can draw (or imagine) surfaces on which the potential V has the same value. Such a surface is called an **equipotential**. Simple examples are shown in Fig. 3.3; for a point charge the equipotentials are spherical surfaces surrounding the charge. For the parallel charged plates (where the potential is proportional to z between the plates) the equipotentials are parallel planes.

A less trivial example is given in Fig. 3.4, where we see the profiles of the equipotential surfaces for the electric dipole.

In general the equipotential surfaces are *perpendicular to the electric field lines.*

The relation between the electric field $\mathbf{E}$ and the potential V can be expressed using the equipotentials and field lines, as illustrated in Fig. 3.5. If we consider a small displacement

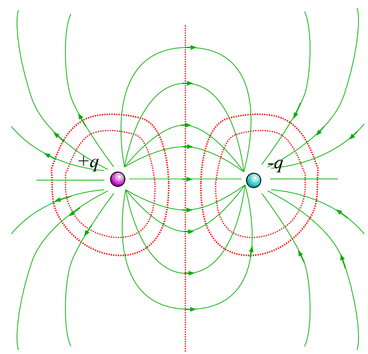

Figure 3.4: Equipotential surfaces for the electric dipole.

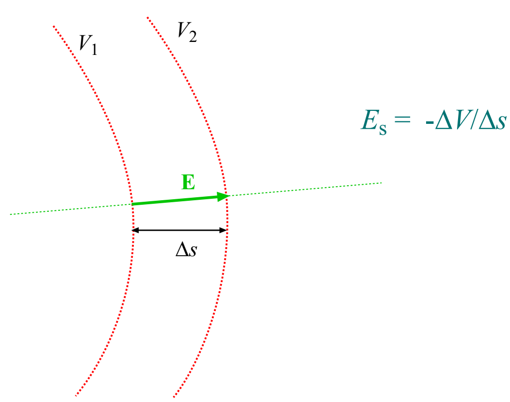

Figure 3.5: Relation between **E** and V. If we go from equipotential V_1 to equipotential V_2 along a field line (perpendicular to the equipotentials, along a coordinate s) then the component of the **E** field in this direction is $E_s = -\Delta V/\Delta s$. We are assuming that Δs is very small.

Δs along a field line, and the displacement takes you from potential V_1 to potential V_2, then the component of the **E** field in the direction of s is given by

$$E_s = -\frac{\Delta V}{\Delta s} \tag{3.8}$$

There's a minus sign here because the electric field always points from higher to lower electric potential; when ΔV is negative, E_s is positive.

When the E field is expressed as in Eq. 3.8 it is clear that the units of the E field can also be given as $\frac{\mathrm{V}}{\mathrm{m}}$ (volts per meter). This *is* exactly the same as the units we had been using, $\frac{\mathrm{N}}{\mathrm{C}}$ (newtons per coulomb).

3.1.4 Capacitance

We've seen one example of a capacitor already (the parallel plates); in general a **capacitor** is a pair of conductors on which we intend to place opposite charges $\pm q$. (To be brief we will say that the charge on a capacitor is "q".)

When the two plates of the capacitor are charged there will be a potential difference ΔV between them (with the positively charged plate at the higher potential). Again, to be brief we will just say that the potential difference of the capacitor is "V".

As one might expect, the greater the charge placed on the plates of the capacitor, the greater the potential difference. In fact, it turns out that they are always *proportional* and the constant of proportionality is called the **capacitance** C of the capacitor:

$$q = CV \tag{3.9}$$

From Eq. 3.9 (which gives $C = \frac{q}{V}$), the units of capacitance have to be C/V. We define this combination of units as the **farad**:

$$1\,\mathrm{farad} = 1\,\mathrm{F} = 1\,\frac{\mathrm{C}}{\mathrm{V}} \tag{3.10}$$

A farad is actually quite a large amount of capacitance; more commonly one sees capacitors with capacitances on the order of mF or μF.

The constant ϵ_0 is often expressed in terms of this unit:

$$\epsilon_0 = 8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N{\cdot}m}^2} = 8.85 \times 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}}$$

We can work out the capacitance of two parallel plates (which are separated by a "small" distance) using this definition and our earlier results. If the plates of the capacitor have area A and are given a charge q then the charge density on the plates (assumed uniform) is $\sigma = \frac{q}{A}$. Then the magnitude of the E field between the plates is

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$$

If the spacing between the plates is d then putting $\Delta z = d$ in Eq 3.7 gives the magnitude of the potential difference, $V = \Delta V = Ed$, or $E = V/d$. Substituting, this gives

$$\frac{V}{d} = \frac{q}{\epsilon_0 A} \qquad \Longrightarrow \qquad q = \left(\frac{\epsilon_0 A}{d}\right) V$$

Comparing this with Eq. 3.9 gives

$$C = \frac{\epsilon_0 A}{d} \tag{3.11}$$

3.1.5 Dielectrics

The formula given above for the capacitance of a parallel-plate capacitor assumes that there is nothing (except for air, which has a small effect) between the plates. If the volume between the plates is filled with an insulating material (a **dielectric**), then the plates can still store charges but the capacitance needs a correction factor from the value given in Eq. 3.11 (which we now call C_0 for clarity). In general the capacitance will be larger than the air-filled value by a factor of κ:

$$C = \kappa C_0 = \frac{\kappa \epsilon_0 A}{d} \tag{3.12}$$

The (unitless) number κ is characteristic of the substance we put between the plates. Some examples are:

$$\kappa_{\rm Teflon} = 2.1 \qquad \kappa_{\rm Mica} = 5.4 \qquad \kappa_{\rm Water} = 80.4$$

With a dielectric between the plates the electric field is still related to V by $E = V/d$ (because the electric field is still uniform) but it is related to the charges on the plates by:

$$E = \frac{\sigma}{\kappa \epsilon_0} = \frac{q}{\kappa \epsilon_0 A} \tag{3.13}$$

so if we keep the *charge* on the plates the same (as would happen if the capacitor were isolated) then the electric field *decreases* from the value it has without the dielectric in place.

3.1.6 Capacitors and Energy

In putting charge onto the plates of a capacitor one must do *work* in transferring charge from one plate to the other. Thus a capacitor stores energy; if the potential difference of a capacitor is V, the energy stored in the capacitor is given by

$$E = \tfrac{1}{2} C V^2 \tag{3.14}$$

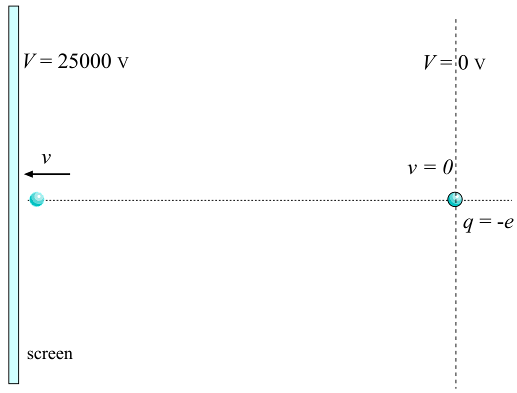

Figure 3.6: Electron in Example 1 moves through a potential difference and gains speed.

If we use Eq. 3.9, we can write this in terms of the charge q on the capacitor,

$$E = \frac{q^2}{2C} \tag{3.15}$$

3.2 Worked Examples

3.2.1 Electric Potential Energy and Electric Potential

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1. In a television tube, electrons strike the screen after being accelerated through a potential difference of $25000\,\mathrm{V}$. The speeds of the electrons are quite large, and for accurate calculations of the speeds, the effects of special relativity must be taken into account. Ignoring such effects, find the electron speed just before the electron strikes the screen. [CJ6 19-5]

The problem is diagrammed in Fig. 3.6. Initially the electron is at a potential $V = 0$ and its speed is zero. Later it is at the screen where the potential is $+25000\,\mathrm{V}$ and its speed is v. In moving toward the screen the kinetic energy of the electron increases and its potential energy decreases such that the *total* energy change of the electron is zero:

$$\Delta\mathrm{PE} + \Delta\mathrm{KE} = 0$$

The change in potential energy of the electron is

$$\Delta\mathrm{PE} = q\Delta V = (-e)\Delta V$$

The change in kinetic energy of the electron is

$$\Delta \mathrm{KE} = \tfrac{1}{2}m_{\mathrm{e}}v^2 - 0 = \tfrac{1}{2}m_{\mathrm{e}}v^2$$

Put these together and get

$$(-e)\Delta V + \tfrac{1}{2}mv^2 = 0 \qquad \Longrightarrow \qquad v^2 = \frac{2e\Delta V}{m_{\mathrm{e}}}$$

Plug in the numbers:

$$v^2 = \frac{2(1.60 \times 10^{-19}\,\mathrm{C})(25000\,\mathrm{V})}{(9.11 \times 10^{-31}\,\mathrm{kg})} = 8.8 \times 10^{15}\,\tfrac{\mathrm{m}^2}{\mathrm{s}^2}$$

And then:

$$v = 9.4 \times 10^7\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

(This is about one-third the speed of light so the caution about the need for relativity was appropriate!)

3.2.2 Calculating the Electric Potential

---

2. Two point charges, $3.40\,\mu\mathrm{C}$ and $-6.10\,\mu\mathrm{C}$ are separated by $1.20\,\mathrm{m}$. What is the electric potential midway between them? [CJ6 19-12]

At the point midway between the charges the distance to *each* charge is 0.60 m. Use Eq. 3.6 to get the electric potential due to this set of point charges:

$$\begin{aligned} V &= k\frac{q_1}{r_1} + k\frac{q_2}{r_2} + \cdots \\ &= (8.99 \times 10^9\,\tfrac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}) \left(\frac{(3.40 \times 10^{-6}\,\mathrm{C})}{(0.60\,\mathrm{m})} + \frac{(-6.10 \times 10^{-6}\,\mathrm{C})}{(0.60\,\mathrm{m})} \right) = -4.05 \times 10^4\,\mathrm{V} \end{aligned}$$

The potential at the given point is $-4.05 \times 10^4\,\mathrm{V}$.

---

3. Oppositely charged plates are separated by $5.33\,\mathrm{mm}$. A potential difference of $600\,\mathrm{V}$ exists between the plates. (a) What is the magnitude of the electric field between the plates? (b) What is the magnitude of the force on an electron between the plates? (c) How much work must be done on the electron to move it to the negative plate if it is initially positioned $2.90\,\mathrm{mm}$ from the positive plate? [SF7 16-7]

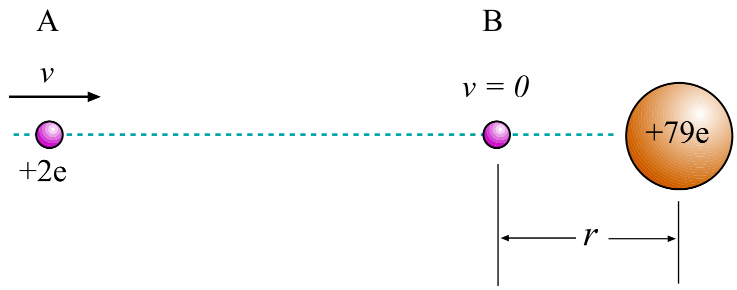

Figure 3.7: Alpha particle (with charge $+2e$) is fired at gold nucleus (with charge $+79e$) in Example 4. At position A it is *very* far away from the gold nucleus and has speed v. At position B, at a distance r from the nucleus, it has come to rest and is turning around.

(a) The electric field is uniform between the plates and we can its magnitude from Eq. 3.7 which gave

$$E = \frac{\Delta V}{d} = \frac{(600\,\mathrm{V})}{(5.33 \times 10^{-3}\,\mathrm{m})} = 1.13 \times 10^{5}\,\frac{\mathrm{V}}{\mathrm{m}}$$

(b) The force on the electron can be found from $\mathbf{F} = q\mathbf{E}$. We have the magnitude of the E field from (a), so the magnitude of the force on an electron is

$$F = |qE| = (1.60 \times 10^{-19}\,\mathrm{C})(1.13 \times 10^{5}\,\tfrac{\mathrm{N}}{\mathrm{C}}) = 1.80 \times 10^{-14}\,\mathrm{N}$$

(c) To push the electron back to the negative plate we must oppose the force found in part (b). Since it is 2.90 mm from the *positive* plate, we need to push the electron a distance

$$s = 5.33\,\mathrm{mm} - 2.90\,\mathrm{mm} = 2.43\,\mathrm{mm}$$

The distance Thus we push with a force of $1.80 \times 10^{-14}\,\mathrm{N}$ for a distance of 2.43 mm. Then the work done is

$$W = Fs = (1.80 \times 10^{-14}\,\mathrm{N})(2.90 \times 10^{-3}\,\mathrm{m}) = 4.38 \times 10^{-17}\,\mathrm{J}$$

So the work done is 4.38×10^{-14} J.

---

4. In Rutherford's famous scattering experiments that led to the planetary model of the atom, alpha particles (having charges of $+2e$ and masses of $6.64 \times 10^{-27}\,\mathrm{kg}$) were fired toward a gold nucleus with charge $+79e$. An alpha particle, initially very far from the gold nucleus, is fired at $2.00 \times 10^{7}\,\frac{\mathrm{m}}{\mathrm{s}}$, as shown in Fig. 3.7. How close does the alpha particle get to the gold nucleus before turning around? Assume the gold nucleus remains stationary. [SF7 16-19]

Because the electric force is a *conservative* force, the total energy of the alpha particle is conserved as it flies toward the gold nucleus. The total energy is the same at position A and position B in Fig. 3.7 and we can use this to figure out the distance at which the alpha particle momentarily comes to rest.

Consider the energy at position A. The alpha particle has kinetic energy $\frac{1}{2}mv^2$. Now, at *all* distances from the gold nucleus the electric potential is

$$V = k\frac{Q}{r} = k\frac{(79e)}{r}$$

which becomes very small as r becomes very large; we assume position A is so far away from the nucleus that we can ignore the electric potential of the nucleus and so the total energy at A is just the kinetic energy:

$$E_A = \tfrac{1}{2}mv^2$$

where $v = 2.00 \times 10^7\ \frac{\mathrm{m}}{\mathrm{s}}$.

At position B the speed of the alpha particle is zero, so it has no kinetic energy. But now we do have to think about the electric potential energy. At a distance r the electric potential is $k(79e)/r$ and the potential *energy* of the alpha particle is $+2e$ (its charge) times this amount, so

$$\mathrm{PE}_B = k\frac{(2e)(79e)}{r} = k\frac{(158e^2)}{r} = E_B$$

From energy conservation we can equate these two expressions for the energy. We get:

$$\tfrac{1}{2}mv^2 = k\frac{(158e^2)}{r} \qquad \Longrightarrow \qquad r = \frac{2k(158)e^2}{mv^2}$$

Plug in the numbers and get:

$$\begin{aligned} r &= \frac{2(8.99 \times 10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}})(158)(1.60 \times 10^{-19}\ \mathrm{C})^2}{(6.64 \times 10^{-27}\ \mathrm{kg})(2.00 \times 10^7\ \frac{\mathrm{m}}{\mathrm{s}})^2} \\ &= 2.74 \times 10^{-14}\ \mathrm{m} \end{aligned}$$

3.2.3 Capacitance

---

5. What voltage is required to store $7.2 \times 10^{-5}\ \mathrm{C}$ **of charge on the plates of a** $6.0\text{-}\mu\mathrm{F}$ **capacitor?** [CJ6 19-36]

Use equation Eq. 3.9, which relates charge, voltage and capacitance:

$$q = CV \qquad \Longrightarrow \qquad V = \frac{q}{C}$$

Plug in the numbers:

$$V = \frac{(7.2 \times 10^{-5}\,\mathrm{C})}{(6.0 \times 10^{-6}\,\mathrm{F})} = 12.0\,\mathrm{V}$$

We must put a potential difference of 12.0 V across the plates of the capacitor.

---

6. An air–filled capacitor consists of two parallel plates, each with an area of $7.60\,\mathrm{cm}^2$ and separated by a distance of $1.80\,\mathrm{mm}$. If a 20.0-V potential difference is applied to these plates, calculate (a) the electric field between the plates, (b) the capacitance, and (c) the charge on each plate. [SF7 16-25]

(a) In between the plates the field is uniform so if we have the potential difference of the plates and their separation, Eq. 3.8 (or Eq. 3.7) gives the magnitude of the electric field in between the plates:

$$E = \frac{\Delta V}{\Delta z} = \frac{V}{d} = \frac{(20.0\,\mathrm{V})}{(1.80 \times 10^{-3}\,\mathrm{m})} = 1.11 \times 10^{4}\,\frac{\mathrm{V}}{\mathrm{m}}$$

(b) To get the capacitance of the parallel plates, use Eq. 3.11, (be careful to convert the units of the area properly...),

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}})(7.60 \times 10^{-4}\,\mathrm{m}^2)}{(1.80 \times 10^{-3}\,\mathrm{m})} = 3.74 \times 10^{-12}\,\mathrm{F}$$

The answer can also be expressed as

$$C = 3.74\,\mathrm{pF}$$

(c) Having the capacitance and potential difference we can get the charge on each plate from Eq. 3.9,

$$\begin{aligned} q &= CV = (3.74 \times 10^{-12}\,\mathrm{F})(20.0\,\mathrm{V}) = 7.47 \times 10^{-11}\,\mathrm{C} \\ &= 74.7 \times 10^{-12}\,\mathrm{C} = 74.7\,\mathrm{pC} \end{aligned}$$

which means that one plate has a charge of $+74.7\,\mathrm{pC}$ and the other has a charge of $-74.7\,\mathrm{pC}$.

3.2.4 Dielectrics

---

7. A parallel plate capacitor has a capacitance of $7.0\,\mu\mathrm{F}$ when filled with a dielectric. The area of each plate is $1.5\,\mathrm{m}^2$ and the separation between the plates is $1.0 \times 10^{-5}\,\mathrm{m}$. What is the dielectric constant of the dielectric? [CJ6 19-35]

Eq. 3.12 gives the capacitance of a parallel plate capacitor when it is filled with dielectric of constant κ. Solving for κ gives

$$\kappa = \frac{dC}{\epsilon_0 A}$$

Plug in the numbers:

$$\kappa = \frac{(1.0 \times 10^{-5}\,\mathrm{m})(7.0 \times 10^{-6}\,\mathrm{F})}{(8.85 \times 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}})(1.5\,\mathrm{m}^2)} = 5.3$$

3.2.5 Capacitors and Energy

---

8. A parallel-plate capacitor has $2.00\,\mathrm{cm}^2$ plates that are separated by $5.00\,\mathrm{mm}$ with air between them. If a 12.0-V battery is connected to this capacitor, how much energy does it store? [SF7 16-43]

Using Eq. 3.11 we find the capacitance of this capacitor (note that $2.00\,\mathrm{cm}^2$ converts to $2.00 \times 10^{-4}\,\mathrm{m}^2$):

$$C = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}\,\tfrac{\mathrm{C}^2}{\mathrm{N \cdot m}^2}) \frac{(2.00 \times 10^{-4}\,\mathrm{m}^2)}{(5.00 \times 10^{-3}\,\mathrm{m})} = 3.54 \times 10^{-13}\,\mathrm{F}$$

Using Eq 3.14 we find the energy stored in this capacitor:

$$\text{Energy} = \tfrac{1}{2} C V^2 = \tfrac{1}{2}(3.54 \times 10^{-13}\,\mathrm{F})(12.0\,\mathrm{V})^2 = 2.55 \times 10^{-11}\,\mathrm{J}$$

Chapter 4

Electric Current and Resistance

4.1 The Important Stuff

4.1.1 Electric Current

The results of the last three chapters (particularly those involving conductors) apply to the special case that electric charges are not in motion, the electrostatic case. For that case, all the points of a single conductor were at the same potential and the electric field was zero within the material of the conductor.

In certain situations we can maintain the motion of charges through a conductor, as when we connect a battery across the ends of a wire. In that case electric charge (negative charge, as it turns out) moves through the wire and there will be potential differences between the points of the conductor.

Even though it is the electrons in the material which move it turns out that it makes no difference if we think of positive charges moving in the opposite direction, and that is how we will think of current.

For a long conductor the fact that electric charge doesn't build up anywhere implies that the amount of charge per time passing any point is the same. For clarity, we can imagine a plane cutting through a conductor as shown in Fig. 4.1. We imagine counting the charge per time which crosses this plane, and the amount of charge per time is the **current** I,

$$I = \frac{\Delta q}{\Delta t} \tag{4.1}$$

Electric current (as we will use it) is a scalar and from Eq. 4.1 must have units of C/s. We define this combination of units to be the **ampere**,

$$1\,\mathrm{ampere} = 1\,\mathrm{A} = 1\,\tfrac{\mathrm{C}}{\mathrm{s}} \tag{4.2}$$

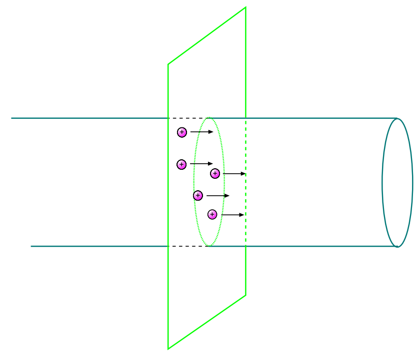

Figure 4.1: Electric current: Charge per time passing through a cross-section of a wire.

Electric current can be related to the number density of electrons in a conductor and the speed with which they move by:

$$I = nqv_d A \tag{4.3}$$

where n is the number density of charge carriers (electrons, usually), q is the value of their charge, v_d is the **drift velocity**, the speed with which the carriers actually move in the wire (on average) and A is the cross-sectional area of the wire.

4.1.2 Ohm's Law

For many substances it is found that the current flowing through a wire made of the material is proportional to the potential difference across its ends: $I \propto V$. We write this relation in the following way:

$$\frac{V}{I} = R \qquad \text{or} \qquad V = IR \tag{4.4}$$

where R is constant which depends on the properties of the wire (its material and its dimensions). R is called the **resistance** of the wire and relation 4.4 is known as **Ohm's law**. It is really an *empirical* relation, i.e. one which does not come directly from the laws of physics but which is obeyed pretty well in the real world and is very useful.

From the relation $R = V/I$ we see that the units of resistance must be $\frac{\mathrm{V}}{\mathrm{A}}$. This combination of units is called an **ohm**:

$$1\,\mathrm{ohm} = 1\,\Omega = 1\,\frac{\mathrm{V}}{\mathrm{A}} \tag{4.5}$$

4.1.3 Resistance and Resistivity

The resistance of a piece of material depends on the type and shape of the material. If the piece has length L and cross-sectional area A, the resistance is

$$R = \rho \frac{L}{A} \tag{4.6}$$

where ρ is a constant (for a given material at a given temperature) known as the **resistivity** of the material. Some selected values for ρ are:

$$\rho_{\mathrm{Copper}} = 1.72 \times 10^{-8}\,\Omega\cdot\mathrm{m} \qquad \rho_{\mathrm{Aluminum}} = 2.82 \times 10^{-8}\,\Omega\cdot\mathrm{m} \qquad \rho_{\mathrm{Carbon}} = 3.5 \times 10^{-5}\,\Omega\cdot\mathrm{m}$$

The resistivity of a material usually increases with temperature. It generally follows an empirical formula given by:

$$\rho = \rho_0[1 + \alpha(T - T_0)] \tag{4.7}$$

where ρ and ρ_0 are the resistivities of the material at temperatures T and T_0, respectively. The constant α is the **temperature coefficient of resistivity**.

4.1.4 Electric Power

As charge moves through the wires of an electric circuit, they lose electric potential energy. (When charge Δq moves through a potential difference V, it loses ΔqV of potential energy.) The *rate* of energy loss is the power P delivered to the circuit elements,

$$P = \frac{\Delta qV}{\Delta t} = \frac{\Delta q}{\Delta t}V = IV$$

that is,

$$P = IV \tag{4.8}$$

Electric power is measured in joules per second, or watts: $1\,\frac{\mathrm{J}}{\mathrm{s}} = 1\,\mathrm{W}$. (We have already met this unit when we considered mechanical work done per unit time in first-semester physics.)

The energy goes into heating the resistor.

Using Ohm's law, ($V = IR$, or $I = V/R$) we can show that the power deliver to a circuit element of resistance R can also be written as

$$P = I^2R \qquad \text{or} \qquad P = \frac{V^2}{R} \tag{4.9}$$

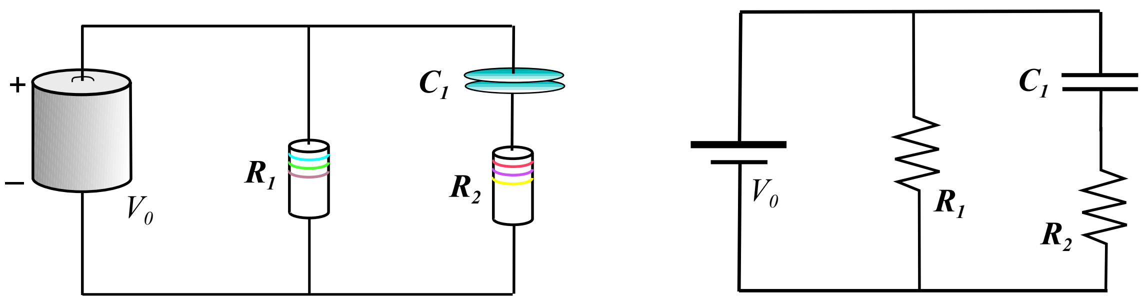

Figure 4.2: (a) Battery connected to two resistors and a capacitor. (b) Schematic diagram for this circuit.

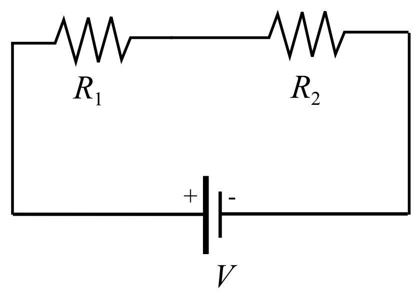

Figure 4.3: Circuit with two resistors in series.

4.1.5 Series and Parallel Circuits

We will now consider circuits which are more complicated than a single battery connected to a single resistor. To make progress we will need to use **schematic diagrams** as shown in Fig. 4.2. In these diagrams a battery is represented by two parallel lines; the longer line represents the positive end of the battery (the one at the higher potential). A resistor is represented by a zigzag line and a capacitor is represented by two parallel lines.

In any of these circuits, the precise shapes of the wires which connect the elements does matter; we only need to care about the circuit elements and *how* they are connected to the other elements.

The first kind of circuit we consider is where a battery is connected to two or more resistors which are joined end-to-end. Such a circuit is shown in Fig. 4.3. In this circuit the *same current* I flows through R_1 and R_2 (it has nowhere else to go). From Ohm's law the drops in potential across the two resistors are IR_1 and IR_2. The sum of these potential drops must equal V, the gain in potential across the leads of the battery. So then:

$$IR_1 + IR_2 = V \qquad \Longrightarrow \qquad V = I(R_1 + R_2) = IR_{\rm equiv}$$

where the *equivalent* resistance of the pair is the sum, $R_1 + R_2$.

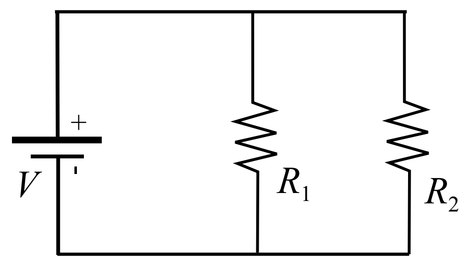

Figure 4.4: Circuit with two resistors in parallel.

This result generalizes to three or more resistors in series, so we have found that when we have a number of resistors in series, then for the purposes of finding the common current through them we can replace them with the equivalent resistance given by

$$R_{\rm ser} = R_1 + R_2 + R_3 + \cdots \tag{4.10}$$

A different arrangement of battery and resistors is shown in Fig. 4.4. Here the end of two resistors are at a comon potential so that the potential drop across the resistors is the same (here, it is V, the battery volatge) but the current through each resistor is *not* the same. If the current through R_1 is I_1 and the current through R_2 is I_2 then Ohm's law gives

$$V = I_1 R_1 = I_2 R_2$$

so that

$$I_1 = \frac{V}{R_1} \qquad \text{and} \qquad I_2 = \frac{V}{R_2}$$

Now if the total current which comes out of the battery is I, then this current splits into the two branches, so that $I = I_1 + I_2$. Combining these results gives

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

We can write this as

$$V = I\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

Now, this looks like Ohm's law where the *equivalent* resistance of the parallel resistors is

$$R_{equiv} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} \qquad \text{or} \qquad \frac{1}{R_{\rm equiv}} = \frac{1}{R_1} + \frac{1}{R_2}$$

So we have an addition rule for resistors in parallel: The *reciprocal* of the equivalent resistance is the sum of the *reciprocals* of the individual resistances. So here it's the *reciprocals* which add together. This rule hold for any number of resistors in parallel so we give the rule for the parallel case as:

$$\frac{1}{R_{\rm par}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \tag{4.11}$$

4.1.6 Kirchhoff's Rules

When analyzing fairly simple circuits the rules for series and parallel resistors along with Ohm's law can give all the currents and potential differences. For more complicated cases— for example networks where there are two or more batteries and resistors with multiple connections— we need more general rules to solve for the currents.

This help is provided by **Kirchhoff's Rules**. These rules are:

Junction Rule: Consider a place where several wires meet— a junction. The sum of the currents going into this junction equals the sum of currents coming out of this junction.

Loop Rule Consider any loop in the circuit. The sum of potential drops equals the sum of potential rises, or more simply with signs properly given to all potential differences, the sum of potential differences is *zero*.

To apply Kirchhoff's Rules to a circuit, assign a current (with magnitude and direction) to each branch in the circuit. Then after choosing a particular loop and a direction in which to go around that loop, use the potential differences given by:

• If you go from the $-$ terminal to the $+$ terminal of a battery of voltage V, the potential difference is $+V$.

• If you go from the $+$ terminal to the $-$ terminal of a battery of voltage V, the potential difference is $-V$.

• If you go across a resistor in the direction of the current I, the potential change is $-IR$ (that is, this is a voltage *drop*).

• If you go across a resistor in the direction *opposite* that of the current I, the potential change is $+IR$ (that is, this is a potential *gain*).

Adding up the potential differences then gives zero for any loop.

4.2 Worked Examples

4.2.1 Electric Current

1. A certain conductor has 7.50×10^{28} free electrons per cubic meter, a cross-sectional area of $4.00 \times 10^{-6}\,\mathrm{m}^2$, and carries a current of $2.50\,\mathrm{A}$. Find the drift speed of the electrons in the conductor. [SF7 17-2]

Use Eq. 4.3 and solve for v_d:

$$I = nqv_dA \qquad \Longrightarrow \qquad v_d = \frac{I}{nqA}$$

Plug in numbers:

$$v_d = \frac{2.50\,\mathrm{C/s}}{(7.50 \times 10^{28}\,\mathrm{m^{-3}})(1.60 \times 10^{-19}\,\mathrm{C})(4.00 \times 10^{-6}\,\mathrm{m^2})} = 5.2 \times 10^{-5}\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

(While this speed may seem implausibly slow, in fact the average drifting motion of the electrons in a wire *is* slow.)

---

2. In a particular television picture tube, the measured beam current is $60.0\,\mu\mathrm{A}$. How many electrons strike the screen every second? [SF7 17-4]

The given current (in amperes, or coulombs per second) gives the *charge* per unit time. We can use the charge of an electron ($1.60 \times 10^{-19}\,\mathrm{C}$, in absolute value) to convert this to *electrons* per unit time:

$$60.0\,\mu\mathrm{A} = (60.0 \times 10^{-6}\,\tfrac{\mathrm{C}}{\mathrm{s}})\left(\frac{1\,\mathrm{electron}}{(1.60 \times 10^{-19}\,\mathrm{C})}\right) = 3.75 \times 10^{14}\,\tfrac{\mathrm{electrons}}{\mathrm{s}}$$

So 3.75×10^{14} electrons hit the screen every second.

4.2.2 Ohm's Law

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3. The filament of a light bulb has a resistance of $580\,\Omega$. A voltage of $120\,\mathrm{V}$ is connected across the filament. How much current is in the filament? [CJ6 20-3]

Ohm's law relates V, I and R; from it, we have $I = V/R$. Plugging in the numbers,

$$I = \frac{V}{R} = \frac{(120\,\mathrm{V})}{(580\,\Omega)} = 0.21\,\mathrm{A}$$

4.2.3 Resistance and Resistivity

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4. A cylindrical copper cable carries a current of $1200\,\mathrm{A}$. There is a potential difference of $1.6 \times 10^{-2}\,\mathrm{V}$ between two points on the cable that are $0.24\,\mathrm{m}$ apart. What is the radius of the cable? [CJ7 20-11]

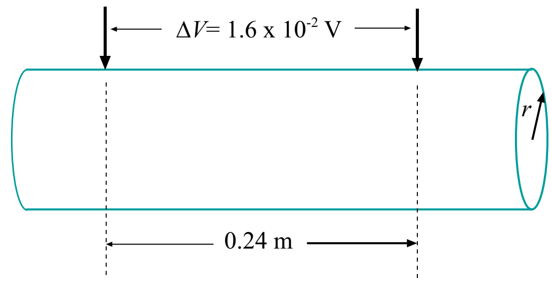

Figure 4.5: Illustration of Example 4

The problem is diagrammed in Fig. 4.5. We have the current in the cable and the potential difference for two different points, so from Ohm's law the resistance of the *part* of the cable between those two points is

$$R = \frac{V}{I} = \frac{(1.6 \times 10^{-2}\,\mathrm{V})}{(1200\,\mathrm{A})} = 1.33 \times 10^{-5}\,\Omega$$

Then from Eq. 4.6, knowing R, L and the resistivity of the material (i.e. copper) we can get the cross-sectional area:

$$R = \rho\frac{L}{A} \qquad \Longrightarrow \qquad A = \frac{\rho L}{R}$$

Plug in the numbers:

$$A = \frac{(1.72 \times 10^{-8}\,\Omega \cdot \mathrm{m})(0.24\,\mathrm{m})}{(1.33 \times 10^{-5}\,\Omega)} = 3.10 \times 10^{-4}\,\mathrm{m}^2$$

The cable has a circular cross-section so that $A = \pi r^2$. Solve for r:

$$r^2 = \frac{A}{\pi} = 9.87 \times 10^{-5}\,\mathrm{m}^2 \qquad \Longrightarrow \qquad r = 9.93 \times 10^{-3}\,\mathrm{m} = 9.93\,\mathrm{mm}$$

---

5. Calculate the diameter of a 2.0**-cm length of tungsten filament in a small lightbulb if its resistance is** $0.050\,\Omega$**.** Use $\rho_{\mathrm{Tung}} = 5.6 \times 10^{-8}\,\Omega \cdot \mathrm{m}$. [SF7 17-13]

We have the resistance of the sample, its length and its resistivity. Use Eq. 4.6 to get the cross-sectional area:

$$R = \rho\frac{L}{A} \qquad \Longrightarrow \qquad \Longrightarrow \qquad A = \frac{\rho L}{R}$$

Use the given values:

$$A = \frac{5.6 \times 10^{-8}\,\Omega \cdot \mathrm{m})(2.0 \times 10^{-2}\,\mathrm{m})}{(0.050\,\Omega)} = 2.2 \times 10^{-8}\,\mathrm{m}^2$$

Then since $A = \pi r^2$ the radius is

$$r^2 = \frac{A}{\pi} = \frac{(2.2 \times 10^{-8}\,\mathrm{m}^2)}{\pi} = 7.1 \times 10^{-9}\,\mathrm{m}^2 \qquad \Longrightarrow \qquad r = 8.4 \times 10^{-5}\,\mathrm{m}$$

and the *diameter* of the wire is

$$d = 2r = 1.7 \times 10^{-4}\,\mathrm{m} = 0.17\,\mathrm{mm}$$

4.2.4 Electric Power

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6. The heating element in an iron has a resistance of $24\,\Omega$. The iron is plugged into a 120-V outlet. What is the power delivered to the iron? [CJ7 20-21]

[We need to bend the rules a bit here; actually a wall outlet delivers an *alternating* voltage, not the constant voltage that we use through this chapter. It turns out that if we treat the given voltage value as constant we do get the right answer.]

Here we are given the potential drop across the resistor (i.e. the iron) and its resistance so that we can one of the equations from 4.9 to get

$$P = \frac{V^2}{R} = \frac{(120\,\mathrm{V})^2}{(24\,\Omega)} = 600\,\mathrm{W}$$

4.2.5 Resistors in Series and in Parallel

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7. A 36.0-Ω resistor and a 18.0-Ω resistor are connected in series across a 15.0-V battery. What is the voltage across (a) the 36.0-Ω resistor and (b) the 18.0-Ω resistor? [CJ7 20-41]

The circuit is shown in Fig. 4.6. As the resistors are in series, the equivalent resisance is the sum of the two:

$$R_{\mathrm{equiv}} = R_1 + R_2 = 36.0\,\Omega + 18.0\,\Omega = 54.0\,\Omega\}$$

and with this value, Ohm's law gives the current I:

$$V = IR \qquad \Longrightarrow \qquad I = \frac{V}{R} = \frac{15.0\,\mathrm{V}}{54.0\,\Omega} = 0.278\,\mathrm{A}$$

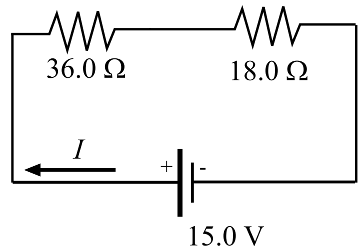

Figure 4.6: Circuit for Example 7.

This is the current which goes through *each* resistor.

Using Ohm's law we can find the voltage (potential difference) across each resistor:

$$V_{36.0} = IR_1 = (0.278\,\mathrm{A})(36.0\,\Omega) = 10.0\,\mathrm{V}$$

$$V_{18.0} = IR_2 = (0.278\,\mathrm{A})(18.0\,\Omega) = 5.00\,\mathrm{V}$$

The sum of the two potential differences is 15.0 V, as it must be since that is the same as potential difference across the terminals of the battery.

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8. What resistance must be placed in parallel with a 155-Ω resistor to make the equivalent resistance 115 Ω? [CJ7 20-48]

Eq. 4.11 gives the equivalent resistance for two resistors in parallel. If one of them is 155 Ω and the equivalent resistance is 115 Ω, then

$$\frac{1}{R_{\mathrm{par}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \qquad \Longrightarrow \qquad \frac{1}{115\,\Omega} = \frac{1}{155\,\Omega} + \frac{1}{R}$$

Solve for R:

$$\frac{1}{R} = \frac{1}{115\,\Omega} - \frac{1}{155\,\Omega} = 2.24 \times 10^{-3}\,\Omega^{-1} \qquad \Longrightarrow \qquad R = 446\,\Omega$$

---

9. Find the equivalent resistance between points a and b in Fig. 4.7. [SF7 18-45]

First we note that the 5.1 Ω and 3.5 Ω resistors are in *series* The picture shows a bend where they join, but that's irrelevant!) They are equivalent to a single resistor of value

$$R_{\mathrm{equiv}} = R_1 + R_2 = 5.1\,\Omega + 3.5\,\Omega = 8.6\,\Omega$$

so we can draw a (new) equvalent circuit as shown in Fig. 4.8(a). The new 8.6 Ω resistor is

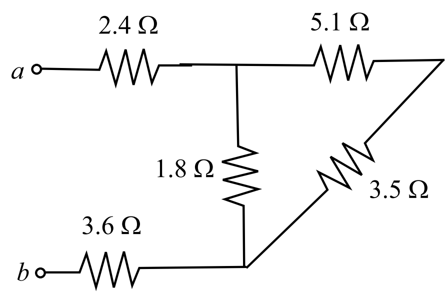

Figure 4.7: Resistor combination for Example 9.

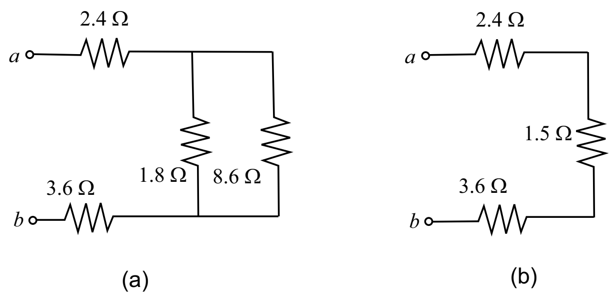

Figure 4.8: Steps in solving Example 9.

in *parallel* with the 1.8 Ω resistor, so their resistances combine as given in Eq. 4.11,

$$\frac{1}{R_{\rm equiv}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{1.8\,\Omega} + \frac{1}{8.6\,\Omega} = 0.672\,\Omega^{-1} \qquad \Longrightarrow \qquad R_{\rm equiv} = 1.5\,\Omega$$

Then replace the parallel resistors in Fig. 4.8(a) with a single 1.5 Ω resistor and we have the circuit shown in Fig. 4.8(b). Here there are three resistors in series so the equivalent resistance is the sum of their values:

$$R_{\rm equiv} = 2.4\,\Omega + 1.5\,\Omega + 3.6\,\Omega = 7.5\,\Omega$$

So the equivalent resistance between points a and b is 7.5 Ω.

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10. (a) Find the equivalent resistance between points a and b in Fig. 4.9. (b) Calculate the current in each resistor if a potential difference of $34.0\,\mathrm{V}$ is applied between points a and b [SF7 18-5]

(a) First, find the equivalent resistance of the pair of parallel resistors in the center. (We have to start with this; we have no other simple series or parallel combination to start with.)

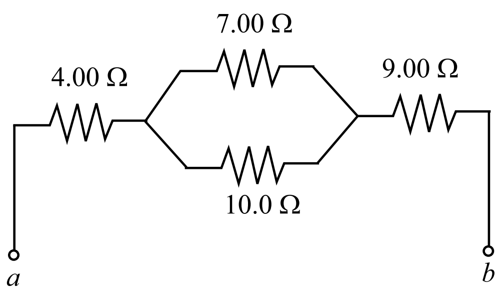

Figure 4.9: Resistor combination for Example 10.

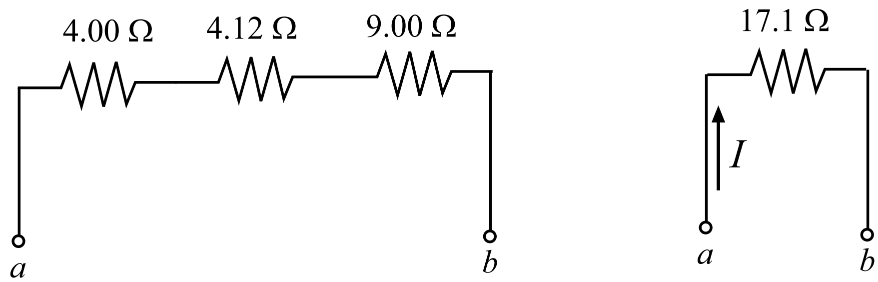

Figure 4.10: First steps in solving Example 10.

Using Eq.‘4.11 we get:

$$\frac{1}{R_{rmequiv}} = \frac{1}{7.00\,\Omega} + \frac{1}{10.0\,\Omega} = 0.243\,\Omega^{-1} \qquad \Longrightarrow \qquad R_{\rm equiv} = 4.12\,\Omega$$

so we can replace this pair by a single $4.12\,\Omega$ resistor; we now have the equivalent circuit shown in Fig. 4.10.

Now the combination *is* a simple series circuit and the equivalent resistance is just the sum of the individual resistances:

$$R_{\rm equiv} = 4.00\,\Omega + 4.12\,\Omega + 9.00\,\Omega = 17.1\,\Omega$$

(b) With the answer to (a) we can get the *total* current I flowing into the network at a and out at b:

$$I = \frac{V}{R_{\rm equiv}} = \frac{(34.0\,\mathrm{V})}{(17.1\,\Omega)} = 1.99\,\mathrm{A}$$

This must be the same as the current in the $4.00\,\Omega$ and $9.00\,\Omega$ resistors.

Using Ohm's law we can find the potential drops across the $4.00\,\Omega$ and $9.00\,\Omega$ resistors. They are

$$V_{4.00} = IR = (1.99\,\mathrm{A})(4.00\,\Omega) = 7.96\,\mathrm{V} \quad \text{and} \quad V_{9.00} = IR = (1.99\,\mathrm{A})(9.00\,\Omega) = 17.9\,\mathrm{V}$$

respectively. But the total drop in potential from a to b is 34.0 V so the drop across the resistor pair must be

$$34.0\,\mathrm{V} - 7.96\,\mathrm{V} - 17.9\,\mathrm{V} = 8.1\,\mathrm{V}$$

and this is the drop in potential of *each* resistor in the pair. Using Ohm's law we get the *current* in each of the resistors:

$$I_{7.00} = \frac{V}{R} = \frac{(8.1\,\mathrm{V})}{(7.00\,\Omega)} = 1.2\,\mathrm{A}$$

$$I_{10.0} = \frac{V}{R} = \frac{(8.1\,\mathrm{V})}{(10.0\,\Omega)} = 0.81\,\mathrm{A}$$

Then the currents are

$$I_{4.00} = 1.99\,\mathrm{A} \qquad I_{7.00} = 1.2\,\mathrm{A} \qquad I_{10.0} = 0.81\,\mathrm{A} \qquad I_{9.00} = 1.99\,\mathrm{A}$$