Chapter 5

Capacitance and Dielectrics

5.1 The Important Stuff

5.1.1 Capacitance

Electrical energy can be stored by putting opposite charges $\pm q$ on a pair of isolated conductors. Being conductors, the respective surfaces of these two objects are all at the same potential so that it makes sense to speak of a potential difference V *between the two conductors*, though one should really write ΔV for this. (Also, we will usually just talk about "the charge q" of the conductor pair though we really mean $\pm q$.)

Such a device is called a **capacitor**. The general case is shown in Fig. 5.1(a). A particular geometry known as the *parallel plate capacitor* is shown in Fig. 5.1(b).

It so happens that if we don't change the configuration of the two conductors, the charge q is proportional to the potential difference V. The proportionality constant C is called the **capacitance** of the device. Thus:

$$q = CV \tag{5.1}$$

The SI unit of capacitance is then $1\,\frac{\mathrm{C}}{\mathrm{V}}$, a combination which is called the **farad**[1]. Thus:

$$1\,\mathrm{farad} = 1\,\mathrm{F} = 1\,\frac{\mathrm{C}}{\mathrm{V}} \tag{5.2}$$

The permittivity constant can be expressed in terms of this new unit as:

$$\epsilon_0 = 8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N{\cdot}m}^2} = 8.85 \times 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}} \tag{5.3}$$

5.1.2 Calculating Capacitance

For various simple geometries for the pair of conductors we can find expressions for the capacitance.

- **Parallel-Plate Capacitor**

[1]Named in honor of the…uh…Austrian physicist Jim Farad (1602–1796) who did some electrical experiments in…um…Berlin. That's it, Berlin.

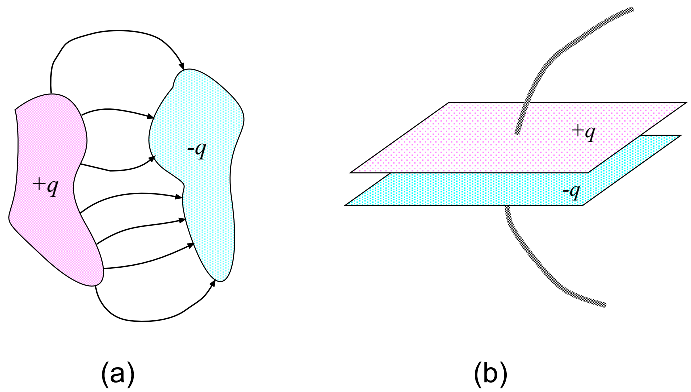

Figure 5.1: (a) Two isolated conductors carrying charges $\pm q$: A capacitor! (b) A more common configuration of conductors for a capacitor: Two isolated parallel conducting sheets of area A, separated by (small) distance d.

The most common geometry we encounter is one where the two conductors are parallel plates (as in Fig. 5.1(b), with the stipulation that the dimensions of the plates are "large" compared to their separation to minimize the "fringing effect".

For a parallel-plate capacitor with plates of area A separated by distance d, the capacitance is given by

$$C = \frac{\epsilon_0 A}{d} \tag{5.4}$$

- **Cylindrical Capacitor**

In this geometry there are two coaxial cylinders where the radius of the inner conductor is a and the inner radius of the outer conductor is b. The length of the cylinders is L; we stipulate that L is large compared to b.

For this geometry the capacitance is given by

$$C = 2\pi\epsilon_0 \frac{L}{\ln(b/a)} \tag{5.5}$$

- **Spherical Capacitor**

In this geometry there are two concentric spheres where the radius of the inner sphere is a and the inner radius of the outer sphere is b. For this geometry the capacitance is given by:

$$C = 4\pi\epsilon_0 \frac{ab}{b-a} \tag{5.6}$$

5.1.3 Capacitors in Parallel and in Series

• **Parallel Combination:** Fig. 5.2 shows a configuration where three capacitors are combined in **parallel** across the terminals of a battery. The battery gives a constant potential

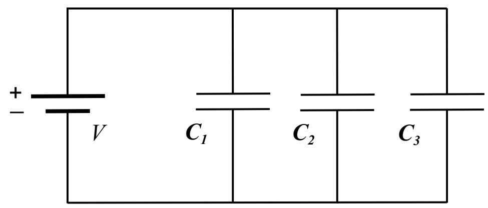

Figure 5.2: Three capacitors are combined in *parallel* across a potential difference V (produced by a battery).

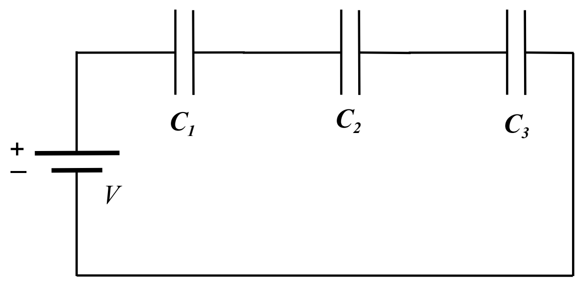

Figure 5.3: Three capacitors are combined in *series* across a potential difference V (produced by a battery).

difference V across the plates of *each* of the capacitors. The charges q_1, q_2 and q_3 which collect on the plates of the respective capacitors are *not* the same, but will be found from

$$q_1 = c_1 V \qquad q_2 = C_2 V \qquad q_3 = C_3 V \ .$$

The total charge on the plates, $q = q_1 + q_2 + q_3$ is related to the potential difference V by $q = C_{\rm equiv} V$, where $C_{\rm equiv}$ is the equivalent capacitance of the combination. In general, the equivalent capacitance for a set of capacitors which are in parallel is given by

$$C_{\rm equiv} = \sum_i C_i \qquad \textbf{Parallel} \tag{5.7}$$

• **Series Combination:** Fig. 5.3 shows a configuration where three capacitors are combined in **series** across the terminals of a battery. Here the charges which collect on the respective capacitor plates *are* the same (q) but the potential differences across the capacitors are *different*. These potential differences can be found from

$$V_1 = \frac{q}{C_1} \qquad V_2 = \frac{q}{C_2} \qquad V_3 = \frac{q}{C_3}$$

where the individual potential differences add up to give the total: $V_1 + V_2 + V_3 = V$. In general, the effective capacitance for a set of capacitors which are in series is

$$\frac{1}{C_{\rm equiv}} = \sum_i \frac{1}{C_i} \qquad \textbf{Series} \tag{5.8}$$

5.1.4 Energy Stored in a Capacitor

When we consider the work required to charge up a capacitor by moving a charge $-q$ from on plate to another we arrive at the potential energy U of the charges, which we can view as the energy stored in the electric field between the plates of the capacitor. This energy is:

$$U = \frac{q^2}{2C} = \tfrac{1}{2}CV^2 \tag{5.9}$$

If we associate the energy in Eq. 5.9 with the region where there *is* any electric field, the interior of the capacitor (the field is effectively zero outside) then we arrive at an energy per unit volume for the electric field, i.e. an **energy density**, u. It is:

$$u = \tfrac{1}{2}\epsilon_0 E^2 \tag{5.10}$$

This result also holds for *any* electric field, regardless of its source.

5.1.5 Capacitors and Dielectrics

If we fill the region between the plates of a capacitor with an insulating material the capacitance will be increased by some numerical factor κ:

$$C = \kappa C_{\rm air} \quad . \tag{5.11}$$

The number κ (which is unitless) is called the **dielectric constant** of the insulating material.

5.2 Worked Examples

5.2.1 Capacitance

1. Show that the two sets of units given for ϵ_0 in Eq. 5.3 are in fact the same.

Start with the new units for ϵ_0, $\frac{\rm F}{\rm m}$. From Eq. 5.2 we substitute $1\,{\rm F} = 1\,\frac{\rm C}{\rm V}$ so that

$$1\,\frac{\rm F}{\rm m} = 1\,\frac{\rm C/V}{\rm m} = 1\,\frac{\rm C}{\rm V\cdot m}$$

Now use the definition of the volt from Eq. 4.4: $1\,{\rm V} = 1\,{\rm J/C} = 1\,{\rm N\cdot m/C}$ to get

$$1\,\frac{\rm F}{\rm m} = 1\,\frac{\rm C}{\frac{\rm N\cdot m}{\rm C}\cdot {\rm m}} = 1\,\frac{\rm C^2}{\rm N\cdot m^2}$$

So we arrive at the original units of ϵ_0 given in Eq. 1.4.

2. The capacitor shown in Fig. 5.4 has capacitance $25\,\mu{\rm F}$ and is initially un-

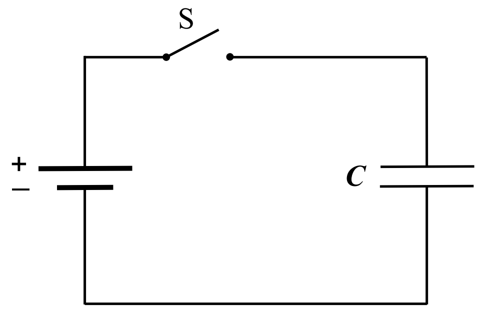

Figure 5.4: Battery and capacitor for Example 2.

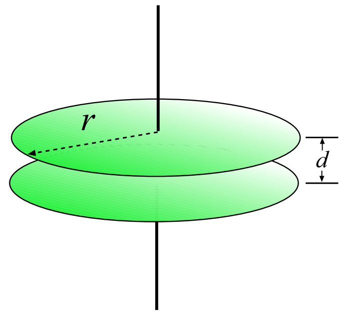

Figure 5.5: Capacitor described in Example 3.

charged. The battery provides a potential difference of 120 V. After switch S is closed, how much charge will pass through it?

When the switch is closed, then the charge q which collects on the capacitor plates is given by $q = CV$. Plugging in the given values for the capacitance C and the potential difference V, we find:

$$\begin{aligned} q &= CV = (25 \times 10^{-6}\,\mathrm{F})(120\,\mathrm{V}) \\ &= 3.0 \times 10^{-3}\,\mathrm{C} = 3.0\,\mathrm{mC} \end{aligned}$$

This is the amount of charge which has been exchanged between the top and bottom plates of the capacitor. So 3.0 mC of charge has passed through the switch.

5.2.2 Calculating Capacitance

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3. A parallel–plate capacitor has circular plates of 8.2 cm radius and 1.3 mm separation. (a) Calculate the capacitance. (b) What charge will appear on the plates if a potential difference of 120 V is applied?

(a) The capacitor is illustrated in Fig. 5.5. The area of the plates is $A = \pi r^2$ so that with

$r = 8.2\,\mathrm{cm}$ and $d = 1.3\,\mathrm{mm}$ and using Eq. 5.4 we get:

$$\begin{aligned} C &= \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 \pi r^2}{d} \\ &= \frac{(8.85 \times 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}})\pi(8.2 \times 10^{-2}\,\mathrm{m})^2}{(1.3 \times 10^{-3}\,\mathrm{m})} \\ &= 1.4 \times 10^{-10}\,\mathrm{F} = 140\,\mathrm{pF} \end{aligned}$$

(b) When a potential of 120 V is applied to the plates of the capacitor the charge which appears on the plates is

$$q = CV = (1.4 \times 10^{-10}\,\mathrm{F})(120\,\mathrm{V}) = 1.7 \times 10^{-8}\,\mathrm{C} = 17\,\mathrm{nC}$$

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4. You have two flat metal plates, each of area $1.00\,\mathrm{m}^2$, with which to construct a parallel-plate capacitor. If the capacitance of the device is to be $1.00\,\mathrm{F}$, what must be the separation between the plates? Could this capacitor actually be constructed?

In Eq. 5.4 (formula for C for a parallel-plate capacitor) we have C and A. We can solve for the separation d:

$$C = \frac{\epsilon_0 A}{d} \qquad \Longrightarrow \qquad d = \frac{\epsilon_0 A}{C}$$

Plug in the numbers:

$$d = \frac{(8.85 \times 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}})(1.00\,\mathrm{m}^2)}{(1.00\,\mathrm{F})} = 8.85 \times 10^{-12}\,\mathrm{m}$$

This is an *extremely* tiny length if we are thinking about making an actual device, because the typical "size" of an atom is on the order of $1.0 \times 10^{-10}\,\mathrm{m}$. Our separation d is ten times *smaller* than that, so the atoms in the plates would not be truly separated! So a suitable capacitor could *not* be constructed.

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5. A $2.0 - \mu\mathrm{F}$ spherical capacitor is composed of two metal spheres, one having a radius twice as large as the other. If the region between the spheres is a vacuum, determine the volume of this region.

The capacitance of a ("air–filled") spherical capacitor is

$$C = 4\pi\epsilon_0 \frac{ab}{(b-a)} \quad .$$

where a and b are the radii of the concentric spherical plates. Here we are given that $b = 2a$, so we then have:

$$C = 4\pi\epsilon_0 \frac{2a^2}{a} = 8\pi\epsilon_0 a$$

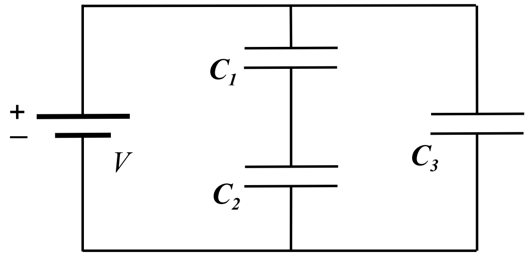

Figure 5.6: Configuration of capacitors for Example 6.

We are given the value of C so we can solve for a:

$$a = \frac{C}{8\pi\epsilon_0} = \frac{(2.0\times 10^{-6}\,\mathrm{F})}{8\pi(8.85\times 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}})} = 9.0\times 10^{3}\,\mathrm{m} \qquad (!)$$

so that $b = 2a = 1.8\times 10^4$ m.

Then the volume of the enclosed region between the two plates is:

$$\begin{aligned} V_{\mathrm{enc}} &= \tfrac{4}{3}\pi b^3 - \tfrac{4}{3}\pi a^3 = \tfrac{4}{3}\pi((2a)^3 - a^3) = \tfrac{4}{3}\pi(7a^3) \\ &= 2.1\times 10^{13}\,\mathrm{m}^3 \end{aligned}$$

5.2.3 Capacitors in Parallel and in Series

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6. In Fig. 5.6, find the equivalent capacitance of the combination. Assume that $C_1 = 10.0\,\mu\mathrm{F}$, $C_2 = 5.00\,\mu\mathrm{F}$, and $C_3 = 4.00\,\mu\mathrm{F}$

The configuration given in the figure is that of a *series* combination of two capacitors (C_1 and C_2) combined in *parallel* with a single capacitor (C_3). We can use the reduction formulae Eq. 5.8 and Eq. 5.7 to give a *single* equivalent capacitance.

First combine the series capacitors with Eq. 5.8. The equivalent capacitance is:

$$\frac{1}{C_{\mathrm{equiv}}} = \frac{1}{10.0\,\mu\mathrm{F}} + \frac{1}{5.00\,\mu\mathrm{F}} = 0.300\,\mu\mathrm{F}^{-1} \qquad \Longrightarrow \qquad C_{\mathrm{equiv}} = 3.33\,\mu\mathrm{F}$$

After this reduction, the configuration is as shown in Fig. 5.7(a). Now we have two capacitors in parallel. By Eq. 5.7 the equivalent capacitance is just the *sum* of the two values:

$$C_{\mathrm{equiv}} = 3.33\,\mu\mathrm{F} + 4.00\,\mu\mathrm{F} = 7.33\,\mu\mathrm{F}$$

The final equivalent capacitance is shown in Fig. 5.7(b).

The equivalent capacitance of the combination is $7.33\,\mu\mathrm{F}$.

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7. How many $1.00\,\mu\mathrm{F}$ capacitors must be connected in parallel to store a charge of $1.00\,\mathrm{C}$ with a potential of $110\,\mathrm{V}$ across the capacitors?

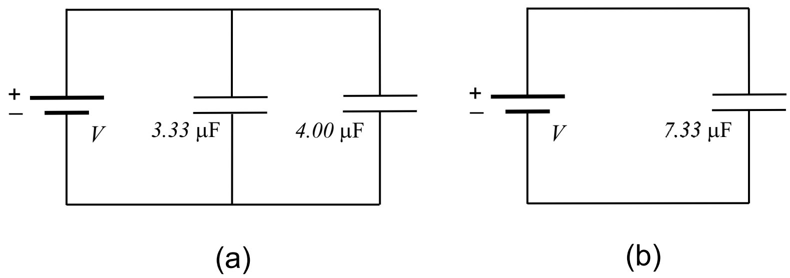

Figure 5.7: (a) Series capacitors in previous figure have been combined as a single equivalent capacitor. (b) Parallel combination in (a) has been combined to give a single equivalent capacitor.

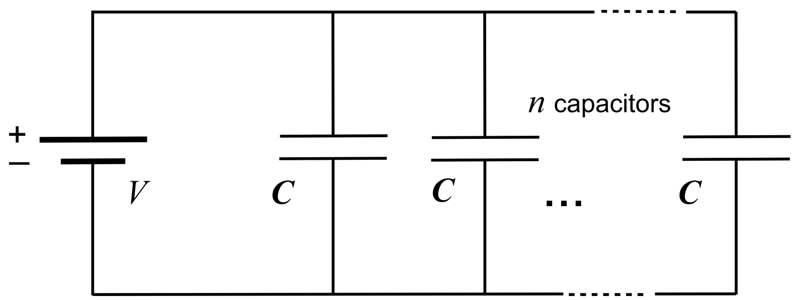

Figure 5.8: n capacitors in parallel, for Example 7.

In this problem we imagine a configuration like that shown in Fig. 5.8, where we have n capacitors with $C = 1.00\,\mu\mathrm{F}$ connected in parallel across a potential difference of $V = 110\,\mathrm{V}$. Since parallel capacitors simply *add* to give the equivalent capacitance (see Eq. 5.7) we have $C_{\mathrm{equiv}} = nC$, and the potential difference across the combination is related to the *total* charge q_{tot} on the plates by $q_{\mathrm{tot}} = C_{\mathrm{equiv}}V = nCV$. We then use this to solve for n:

$$n = \frac{q_{\mathrm{tot}}}{CV} = \frac{(1.00\,\mathrm{C})}{(1.00\times10^{-6}\,\mathrm{F})(110\,\mathrm{V})} = 9.09\times10^{3} \ .$$

So one would need to hook up $n = 9090$ capacitors (!) to store the 1.00 C of charge.

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8. Each of the uncharged capacitors in Fig. 5.9 has a capacitance of $25.0\,\mu\mathrm{F}$. A potential difference of $4200\,\mathrm{V}$ is established when the switch is closed. How many coulombs of charge then pass through the meter A?

The (total) charge which passes through the (current) meter A is the total charge which collects on the plates of the three capacitors. We note that for each capacitor the potential difference across the plates (after the switch is closed) is 4200 V. So the charge on each capacitor is

$$q = CV = (25.0\times10^{-6}\,\mathrm{F})(4200\,\mathrm{V}) = 0.105\,\mathrm{C}$$

and the total charge is

$$q_{\mathrm{Total}} = 3(0.105\,\mathrm{C}) = 0.315\,\mathrm{C} \ .$$

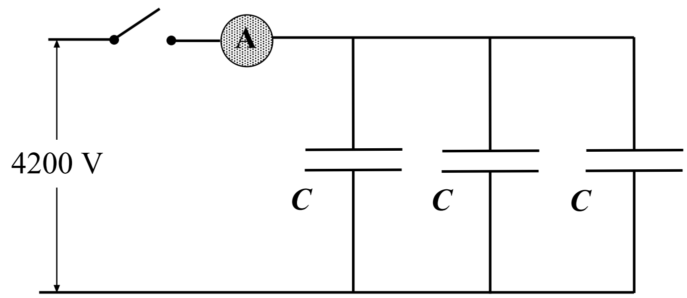

Figure 5.9: Configuration of capacitors for Example 8.

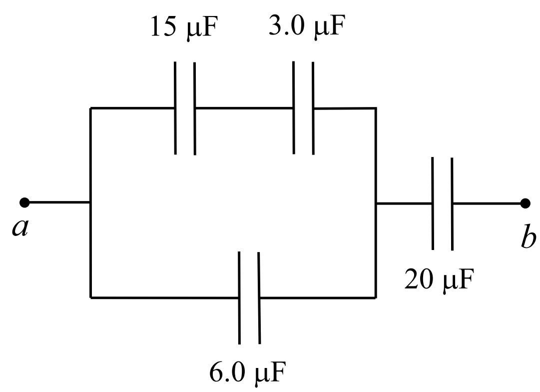

Figure 5.10: Combination of capacitors for Example 9.

So this is the amount of charge which passes through meter A.

We could also note that the equivalent capacitance of the three parallel capacitors is

$$C_{\rm equiv} = 3(25.0\,\mu{\rm F}) = 75.0\,\mu{\rm F}$$

and with 4200 V across the leads of the equivalent capacitance the total charge which collects on the plates is

$$q_{\rm Total} = C_{\rm equiv}V = (75.0\times 10^{-6}\,{\rm F})(4200\,{\rm V}) = 0.315\,{\rm C} \ \ .$$

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9. Four capacitors are connected as shown in Fig. 5.10. (a) Find the equivalent capacitance between points a and b. (b) Calculate the charge on each capacitor if $V_{ab} = 15\,{\rm V}$.

(a) To get the equivalent capacitance of the set of capacitors between a and b: First note that the 15 μF and 3.0 μF capacitors are in series so they combine as:

$$\frac{1}{C_{\rm equiv}} = \frac{1}{15\,\mu{\rm F}} + \frac{1}{3.0\,\mu{\rm F}} = 0.40\,\mu{\rm F}^{-1} \qquad \Longrightarrow \qquad C_{\rm equiv} = 2.5\,\mu{\rm F}$$

After this reduction, the configuration is as shown in Fig. 5.11(a). The reduced circuit now

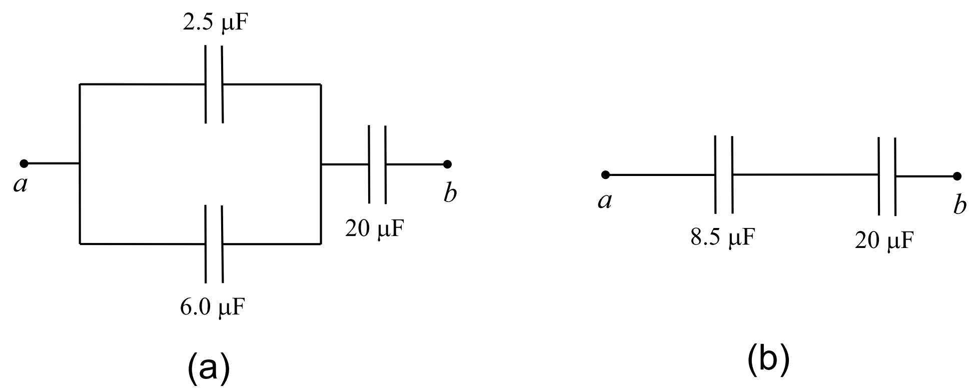

Figure 5.11: (a) After "reduction" of the series pair. (b) After combining two parallel capacitances.

has $2.5\,\mu\text{F}$ and $6.0\,\mu\text{F}$ capacitors in parallel which combine as:

$$C_{\text{equiv}} = 2.5\,\mu\text{F} + 6.0\,\mu\text{F} = 8.5\,\mu\text{F}$$

which gives us the combination shown in 5.11(b).

Finally, the $8.5\,\mu\text{F}$ and $20\,\mu\text{F}$ capacitors in series reduce to:

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{8.5\,\mu\text{F}} + \frac{1}{20\,\mu\text{F}} = 0.168\,\mu\text{F}^{-1} \qquad \Longrightarrow \qquad C_{\text{equiv}} = 5.96\,\mu\text{F}$$

so the equivalent capacitance between points a and b is $5.96\,\mu\text{F}$.

(b) Since the equivalent capacitance between a and b is $5.96\,\mu\text{F}$, the charge which collects on *either end* of the combination is

$$Q = C_{\text{equiv}}V_{ab} = (5.96 \times 10^{-6}\,\text{F})(15\,\text{V} = 8.95 \times 10^{-5}\,\text{C}$$

This is the same as the charge on the far end of the $20\,\mu\text{F}$ capacitor (and thus on either plate of that capacitor) , so we have the charge on that capacitor:

$$Q_{20\,\mu\text{F}} = 8.95 \times 10^{-5}\,\text{C}$$

Now we can find the potential difference across the $20\,\mu\text{F}$ capacitor:

$$V_{20\,\mu\text{F}} = \frac{Q_{20\,\mu\text{F}}}{C_{20\,\mu\text{F}}} = \frac{(8.95 \times 10^{-5}\,\text{C})}{(20 \times 10^{-6}\,\text{F})} = 4.47\,\text{V}$$

With this value, we can find the potential difference between points a and c (see Fig. 5.12):

$$V_{ac} = 15.0\,\text{V} - 4.47\,\text{V} = 10.5\,\text{V}$$

This is now the potential difference across the $6.0\,\mu\text{F}$ capacitor, so we can find its charge:

$$Q_{6.0\,\mu\text{F}} = (6.0 \times 10^{-6}\,\text{F})(10.5\,\text{V}) = 6.32 \times 10^{-5}\,\text{C}$$

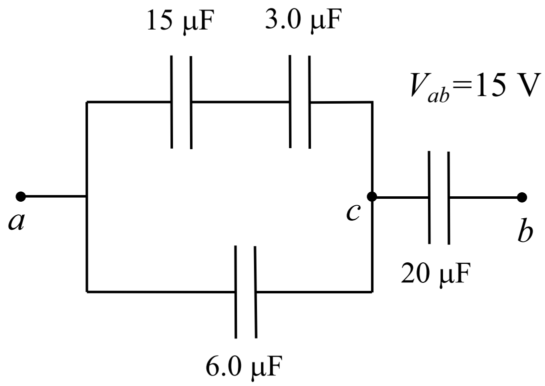

Figure 5.12: Point c comes just before the $20\,\mu$F capacitor. Find V_{ac} by subtracting $V_{20\,\mu\text{F}}$ from 15 V

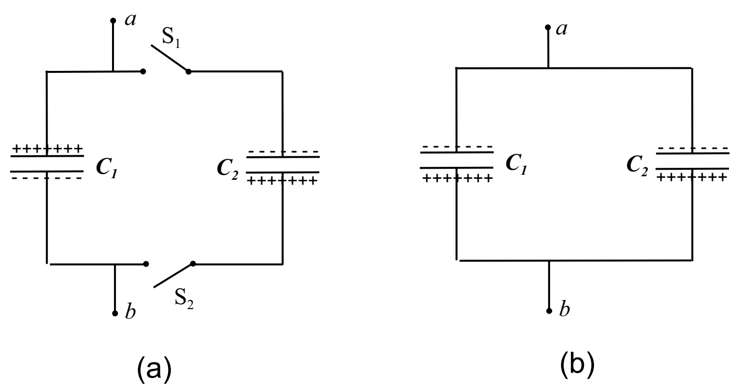

Figure 5.13: Capacitor configuration for Example 10. (a) before switches are closed. (b) After switches are closed, charges redistribute on the plates of C_1 and C_2.

Finally, we note that the potential difference across the $15\,\mu$F — $3.0\,\mu$F series pair is also 10.5 V. Now, the equivalent capacitance of this pair was $2.50\,\mu$F, so that the charge which collects on each end of this combination is

$$Q = C_{\text{equiv}}V = (2.5 \times 10^{-6}\,\text{F})(10.5\,\text{V}) = 2.63 \times 10^{-5}\,\text{C} .$$

But this is the same as the charge on the outer plates of the two capacitors, and that means that both capacitors have the same charge, namely:

$$Q_{15\,\mu\text{F}} = Q_{3.0\,\mu\text{F}} = 2.63 \times 10^{-5}\,\text{C}$$

We now have the charges on all four of the capacitors.

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10. In Fig. 5.13(a), the capacitances are $C_1 = 1.0\,\mu$F and $C_2 = 3.0\,\mu$F and both capacitors are charged to a potential difference of $V = 100\,$V but with opposite polarity as shown. Switches S_1 and S_2 are now closed. (a) What is now the potential difference between a and b? What are now the charges on capacitors (b) 1 and (c) 2?

Let's first find the charges which the capacitors had before the switch was closed. For C_1 the magnitude of its charge was

$$Q_1 = C_1 V = (1.0\,\mu\text{F})(100\,\text{V}) = 1.00 \times 10^{-4}\,\text{C}$$

What we mean here is that the *upper* plate of C_1 had a charge of 1.00×10^{-4} C, because the polarity matters here! So the lower plate of C_1 had a charge of -1.00×10^{-4} C.

For C_2, the *magnitude* of its charge was

$$Q_2 = C_2 V = (3.0\,\mu\text{F})(100\,\text{V}) = 3.00 \times 10^{-4}\,\text{C}$$

but here we mean that the upper plate of C_2 had a charge of -3.00×10^{-4} C because of the polarity indicated in Fig. 5.13(a). So its lower plate had a charge of $+3.00 \times 10^{-4}$ C.

We note that the *total* charge on the upper plates is

$$Q_{1,\,\text{upper}} + Q_{1,\,\text{upper}} = -2.00 \times 10^{-4}\,\text{C}$$

and the total charge on the lower plates is $+2.00 \times 10^{-4}$ C.

Now when the switches are closed the charges on the upper plates will redistribute themselves on the upper plates of C_1 and C_2. Lets call these new charges (on the *upper* plates) Q'_1 and Q'_2. We note that since the total charge on the upper plates was *negative* then it is a net negative charge which shifts around on the upper plates and Q'_1 and Q'_2 are both negative, as indicated in Fig. 5.13(b). By conservation of charge, the total is still equal to -2.00×10^{-4} C:

$$Q'_1 + Q'_2 = -2.00 \times 10^{-4}\,\text{C}$$

Though we don't yet know the new potential difference across each capacitor, we do know that it is the *same* for both. Actually, we know that b must be at the higher potential; we will let the potential change in going from a to b be called V'. Now, the potential for each capacitor is found from $V = Q/C$; actually because of the polarities here (the Q's being negative) we need a minus sign, but the fact that the potential differences are the *same* across both capacitors gives:

$$V' = \frac{-Q'_1}{C_1} = \frac{-Q'_2}{C_2} \qquad \Longrightarrow \qquad Q'_2 = \frac{C_2}{C_1} Q'_1 = \left(\frac{3.0\,\mu\text{F}}{1.0\,\mu\text{F}}\right) Q'_1 = 3.0 Q'_1$$

Substituting this result into the previous one gives

$$Q'_1 + 3.0 Q'_1 = -2.0 \times 10^{-4}\,\text{C} \qquad \Longrightarrow \qquad Q'_1 = \frac{-2.0 \times 10^{-4}\,\text{C}}{4.0} = -5.0 \times 10^{-5}\,\text{C}$$

Having solved for one of the unknowns, we're nearly finished!

The change in potential as we go from a to b is then:

$$V' = \frac{-Q'_1}{C_1} = \frac{+5.0 \times 10^{-5}\,\text{C}}{1.0 \times 10^{-6}\,\text{F}} = 50\,\text{V}$$

(b) The *magnitude* of the new charge on capacitor 1 is $|Q'_1| = 5.0 \times 10^{-5}$ C

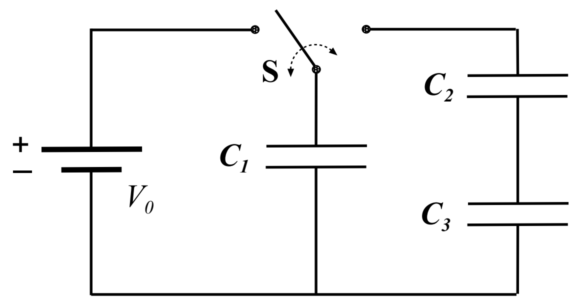

Figure 5.14: Configuration of capacitors and potential difference with switch for Example 11.

(c) Using $Q'_2 = 3.0Q'_1$, the *magnitude* of the new charge on the second capacitor is

$$|Q'_2| = 3.0|Q'_1| = 3.0(5.0 \times 10^{-5}\,\mathrm{C}) = 1.5 \times 10^{-4}\,\mathrm{C} \ .$$

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11. When switch S is thrown to the left in Fig. 5.14, the plates of capacitor 1 acquire a potential difference V_0. Capacitors 2 and 3 are initially uncharged. The switch is now thrown to the right. What are the final charges q_1, q_2 and q_3 on the capacitors?

Initially the only capacitor with a charge is C_1, with a charge given by:

$$q_{1,\,\mathrm{init}} = C_1 V_0 \tag{5.12}$$

since the potential across its plates is V_0.

Now consider what happens when the switch is thrown to the right and the capacitors have charges q_1, q_2 and q_3. Since C_2 and C_3 are joined in series, their charges will be equal, so $q_2 = q_3$ and we only need to find q_2. Also, note that the upper plate of C_1 is only connected to the upper plate of C_2 so that q_1 and q_2 must add up to give the original charge on C_1:

$$q_1 + q_2 = q_{1,\,\mathrm{init}} \tag{5.13}$$

Finally, we note that the potential difference across C_1 is equal to the potential difference across the C_2-C_3 series combination. The equivalent capacitance of the C_2-C_3 combination is:

$$\frac{1}{C_{\mathrm{equiv}}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2 + C_3}{C_2 C_3} \qquad \Longrightarrow \qquad C_{\mathrm{equiv}} = \frac{C_2 C_3}{C_2 + C_3}$$

The potential across C_1 is q_1/C_1, and the potential across the series pair is q_2/C_{equiv}. So equating the potential differences gives

$$\frac{q_1}{C_1} = \frac{q_2}{C_{\mathrm{equiv}}} = \left(\frac{C_2 + C_3}{C_2 C_3}\right) q_2 \tag{5.14}$$

And that's all the equations we need; we can now solve for q_1 and q_2. Eq. 5.14 gives

$$q_2 = \frac{1}{C_1} \frac{C_2 C_3}{C_2 + C_3} q_1 \tag{5.15}$$

and then substitute this and also Eq. 5.12 into Eq. 5.13. We get:

$$q_1 + \frac{1}{C_1}\frac{C_2C_3}{C_2+C_3}q_1 = C_1V_0$$

Factor out q_1 on the left:

$$\left(1 + \frac{1}{C_1}\frac{C_2C_3}{C_2+C_3}\right) q_1 \;=\; \left(\frac{C_1(C_2+C_3)+C_2C_3}{C_1(C_2+C_3)}\right) q_1 \;=\; C_1V_0$$

Now we can isolate q_1:

$$q_1 = \frac{C_1^2(C_2+C_3)}{C_1C_2+C_1C_3+C_2C_3}V_0$$

Then go back and use Eq. 5.15 to get q_2:

$$\begin{aligned} q_2 \;&=\; \frac{1}{C_1}\frac{C_2C_3}{(C_2+C_3)}q_1 \;=\; \frac{1}{C_1}\frac{C_2C_3}{(C_2+C_3)}\frac{C_1^2(C_2+C_3)}{(C_1C_2+C_1C_3+C_2C_3)}V_0 \\ &=\; \frac{C_1C_2C_3}{C_1C_2+C_1C_3+C_2C_3}V_0 \end{aligned}$$

Finally, we recall that $q_3 = q_2$. This gives us expressions for all three charges in terms of the initial parameters.

5.2.4 Energy Stored in a Capacitor

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12. How much energy is stored in one cubic meter of air due to the "fair weather" electric field of magnitude $150\,\mathrm{V/m}$**?**

From Eq. 5.10 we have the energy density of an electric field. (As noted there, the *source* of the electric field is irrelevant.) We get:

$$\begin{aligned} u \;&=\; \tfrac{1}{2}\epsilon_0 E^2 \\ &=\; \tfrac{1}{2}(8.85\times10^{-12}\,\tfrac{\mathrm{C^2}}{\mathrm{N\cdot m^2}})(150\tfrac{\mathrm{V}}{\mathrm{m}})^2 \;=\; 9.96\times10^{-8}\,\tfrac{\mathrm{J}}{\mathrm{m^3}} \end{aligned}$$

So in one cubic meter, 9.96×10^{-8} J of energy are stored.

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13. What capacitance is required to store an energy of $10\,\mathrm{kW\cdot h}$ **at a potential difference of** $1000\,\mathrm{V}$**?**

First, convert the given energy to some sensible units!

$$E \;=\; 10\,\mathrm{kW\cdot h} = 10\times10^3\,\tfrac{\mathrm{J}}{\mathrm{s}}\cdot(1\,\mathrm{h})\left(\frac{3600\,\mathrm{s}}{1\,\mathrm{h}}\right) \;=\; 3.60\times10^7\,\mathrm{J}$$

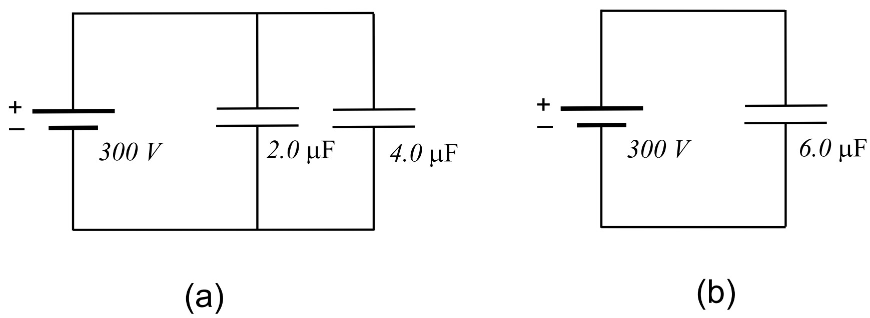

Figure 5.15: (a) Capacitor configuration for Example 14. (b) Equivalent capacitor.

Then use Eq. 5.9 for the energy stored in a capacitor:

$$E = \tfrac{1}{2}CV^2 \qquad \Longrightarrow \qquad C = \frac{2E}{V^2}$$

Plug in the numbers:

$$C = \frac{2(3.60 \times 10^7\,\mathrm{J})}{(1000\,\mathrm{V})^2} = 72\,\mathrm{F}$$

A capacitance of 72 F (big!) is needed.

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14. Two capacitors, of 2.0 and 4.0 μF capacitance, are connected in parallel across a 300 V potential difference. Calculate the total energy stored in the capacitors.

The capacitors and potential difference are diagrammed in Fig. 5.15(a). For the purpose of finding the *total* energy in the capacitors we can replace the two parallel capacitors with a single equivalent capacitor of value 6.0 μF (the original two were in *parallel*, so we *sum* the values). This is because the charge which collects on the equivalent capacitor *is* the sum of charges on the plates of the original two capacitors.

Then the energy stored is

$$E = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(6.0 \times 10^{-6}\,\mathrm{F})(300\,\mathrm{V})^2 = 0.27\,\mathrm{J}$$

Appendix A: Useful Numbers

Conversion Factors

Length	**cm**	**meter**	**km**	**in**	**ft**	**mi**
1 cm =	1	10^{-2}	10^{-5}	0.3937	3.281×10^{-2}	6.214×10^{-6}
1 m =	100	1	10^{-3}	39.37	3.281	6.214×10^{-4}
1 km =	10^5	1000	1	3.937×10^4	3281	06214
1 in =	2.540	2.540×10^{-2}	2.540×10^{-5}	1	8.333×10^{-2}	1.578×10^{-5}
1 ft =	30.48	0.3048	3.048×10^{-4}	12	1	1.894×10^{-4}
1 mi =	1.609×10^5	1609	1.609	6.336×10^4	5280	1

Mass	**g**	**kg**	**slug**	**u**
1 g =	1	0.001	6.852×10^{-2}	6.022×10^{26}
1 kg =	1000	1	6.852×10^{-5}	6.022×10^{23}
1 slug =	1.459×10^4	14.59	1	8.786×10^{27}
1 u =	1.661×10^{-24}	1.661×10^{-27}	1.138×10^{-28}	1

An object with a *weight* of 1 lb has a *mass* of 0.4536 kg.

Constants:

$$
\begin{aligned}
e &= 1.6022 \times 10^{-19}\,\mathrm{C} = 4.8032 \times 10^{-10}\,\mathrm{esu} \\
\epsilon_0 &= 8.85419 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N \cdot m^2}} \\
k &= 1/(4\pi\epsilon_0) = 8.9876 \times 10^9\,\frac{\mathrm{N \cdot m^2}}{\mathrm{C}^2} \\
\mu_0 &= 4\pi \times 10^{-7}\,\frac{\mathrm{N}}{\mathrm{A}^2} = 1.2566 \times 10^{-6}\,\frac{\mathrm{N}}{\mathrm{A}^2} \\
m_{\mathrm{electron}} &= 9.1094 \times 10^{-31}\,\mathrm{kg} \\
m_{\mathrm{proton}} &= 1.6726 \times 10^{-27}\,\mathrm{kg} \\
c &= 2.9979 \times 10^8\,\frac{\mathrm{m}}{\mathrm{s}} \\
N_A &= 6.0221 \times 10^{23}\,\mathrm{mol}^{-1}
\end{aligned}
$$