Chapter 1

Electric Charge; Coulomb's Law

1.1 The Important Stuff

1.1.1 Introduction

During the second semester of your introductory year of physics you will study two special types of forces which occur in nature as a result of the fact that the constituents of matter have electric charge; these forces are the **electric force** and the **magnetic force**. In fact, the study of electromagnetism adds something completely new to the ideas of the mechanics from first semester physics, namely the concept of the electric and magnetic *fields*. These entities are just as real as the masses and forces from first semester and they take center stage when we discuss the phenomenon of electromagnetic radiation, a topic which includes the behavior of visible light.

The entire picture of matter and fields which we will have at the end of this study is known as **classical physics**, but this picture, while complete enough for many fields of engineering, is not a complete statement of the laws of nature (as we now know them). New phenomena which were discovered in the early 20th century demanded revisions in our thinking about the relation of space and time (relativity) and about phenomena on the atomic scale (quantum physics). **Relativity** and **quantum theory** are often known collectively as **modern physics**.

1.1.2 Electric Charge

The phenomenon we recognize as "static electricity" has been known since ancient times. It was later found that there is a physical quantity known as **electric charge** that can be transferred from one object to another. Charged objects can exert forces on other charged objects and also on uncharged objects. Finally, electric charge comes in two types, which we choose to call **positive charge** and **negative charge**.

Substances can be classified in terms of the ease with which charge can move about on their surfaces. **Conductors** are materials in which charges can move about freely; **insulators** are materials in which electric charge is not easily transported.

Electric charge can be measured using the law for the forces between charges (Coulomb's Law). Charge is a *scalar* and is measured in **coulombs** [1]. The coulomb is actually *defined* in terms of electric current (the flow of electrons), which is measured in **amperes**[2]; when the current in a wire is 1 ampere, the amount of charge that flows past a given point in the wire in 1 second is 1 coulomb. Thus,

$$1\,\text{ampere} = 1\,\text{A} = 1\,\tfrac{\text{C}}{\text{s}}\ .$$

As we now know, when charges are transferred by simple interactions (i.e. rubbing), it is a *negative* charge which is transferred, and this charge is in the form of the fundamental particles called **electrons**. The charge of an electron is $1.6022 \times 10^{-19}\,\text{C}$, or, using the definition

$$e = 1.602177 \times 10^{-19}\,\text{C} \tag{1.1}$$

the electron's charge is $-e$. The proton has charge $+e$. The particles found in nature all have charges which are integral multiples of the **elementary charge** e: $q = ne$ where $n = 0, \pm 1, \pm 2 \ldots$. Because of this, we say that charge is **quantized**.

The mass of the electron is

$$m_{\text{e}} = 9.1094 \times 10^{-31}\,\text{kg} \tag{1.2}$$

1.1.3 Coulomb's Law

Coulomb's Law gives the force of attraction or repulsion between two point charges. If two point charges q_1 and q_2 are separated by a distance r then the magnitude of the force of repulsion or attraction between them is

$$F = k\frac{|q_1|\,|q_2|}{r^2} \qquad \text{where} \qquad k = 8.9876 \times 10^9\,\tfrac{\text{N}\cdot\text{m}^2}{\text{C}^2} \tag{1.3}$$

This is the magnitude of the force which *each charge exerts on the other charge* (recall Newton's 3rd law). The symbol k as used here has to do with electrical forces; it has nothing to do with any spring constants or Boltzmann's constant!

If the charges q_1 and q_2 are of the same sign (both positive or both negative) then the force is mutually *repulsive* and the force on each charge points *away from* the other charge. If the charges are of opposite signs (one positive, one negative) then the force is mutually *attractive* and the force on each charge points *toward* the other one. This is illustrated in Fig. 1.1.

The constant k in Eq. 1.3 is often written as

$$k = \frac{1}{4\pi\epsilon_0} \qquad \text{where} \qquad \epsilon_0 = 8.85419 \times 10^{-12}\,\tfrac{\text{C}^2}{\text{N}\cdot\text{m}^2} \tag{1.4}$$

[1] Named in honor of the...uh...Dutch physicist Jim Coulomb (1766–1812) who did some electrical experiments in...um...Paris. That's it, Paris.

[2] Named in honor of the...uh...German physicist Jim Ampere (1802–1807) who did some electrical experiments in...um...Düsseldorf. That's it, Düsseldorf.

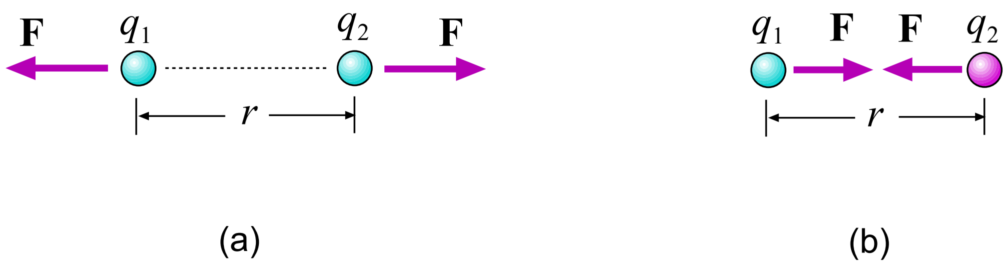

Figure 1.1: (a) Charges q_1 and q_2 have the same sign; electric force is repulsive. (b) Charges q_1 and q_2 have opposite signs; electric force is attractive.

for historical reasons but also because in later applications the constant ϵ_0 is more convenient. ϵ_0 is called the **permittivity constant** [3]

When several points charges are present, the total force on a particular charge q_0 is the *vector sum* of the individual forces gotten from Coulomb's law. (Thus, electric forces have a **superposition** property.) For a continuous distribution of charge we need to divide up the charge distribution into infinitesimal pieces and add up the individual forces with integrals to get the net force.

1.2 Worked Examples

1.2.1 Electric Charge

1. What is the total charge of $75.0\,\mathrm{kg}$ of electrons? [HRW6 22-19]

The mass of *one* electron is 9.11×10^{-31} kg, so that a mass $M = 75.0\,\mathrm{kg}$ contains

$$N = \frac{M}{m_{\mathrm{e}}} = \frac{(75.0\,\mathrm{kg})}{(9.11 \times 10^{-31}\,\mathrm{kg})} = 8.23 \times 10^{31} \quad \text{electrons}$$

The charge of *one* electron is $-e = -1.60 \times 10^{-19}\,\mathrm{C}$, so that the total charge of N electrons is:

$$Q = N(-e) = (8.23 \times 10^{31})(-1.60 \times 10^{-19}\,\mathrm{C}) = -1.32 \times 10^{13}\,\mathrm{C}$$

2. (a) How many electrons would have to be removed from a penny to leave it with a charge of $+1.0 \times 10^{-7}\,\mathrm{C}$? (b) To what fraction of the electrons in the penny does this correspond? [A penny has a mass of $3.11\,\mathrm{g}$; assume it is made entirely of copper.] [HRW6 22-23]

[3] In these notes, k will be used mainly in the first chapter; thereafter, we will make increasing use of ϵ_0!

(a) From Eq. 1.1 we know that as *each* electron is removed the penny picks up a charge of $+1.60 \times 10^{-19}\,\mathrm{C}$. So to be left with the given charge we need to remove N electrons, where N is:

$$N = \frac{q_{\mathrm{Total}}}{q_{\mathrm{e}}} = \frac{(1.0 \times 10^{-7}\,\mathrm{C})}{(1.60 \times 10^{-19}\,\mathrm{C})} = 6.2 \times 10^{11} \ .$$

(b) To answer this part, we will need the total number of electrons in a neutral penny; to find this, we need to find the number of copper atoms in the penny and use the fact that each (neutral) atom contains 29 electrons. To get the *moles* of copper atoms in the penny, divide its mass by the atomic weight of copper:

$$n_{\mathrm{Cu}} = \frac{(3.11\,\mathrm{g})}{(63.54\,\frac{\mathrm{g}}{\mathrm{mol}})} = 4.89 \times 10^{-2}\,\mathrm{mol}$$

The number of copper atoms is

$$N_{\mathrm{Cu}} = n_{\mathrm{Cu}} N_{\mathrm{A}} = (4.89 \times 10^{-2}\,\mathrm{mol})(6.022 \times 10^{23}\,\mathrm{mol}^{-1}) = 2.95 \times 10^{22}$$

and the number of electrons in the penny was (originally) 29 times this number,

$$N_{\mathrm{e}} = 29 N_{\mathrm{Cu}} = 29(2.95 \times 10^{22}) = 8.55 \times 10^{23}$$

so the *fraction* of electrons removed in giving the penny the given electric charge is

$$f = \frac{(6.2 \times 10^{11})}{(8.55 \times 10^{23})} = 7.3 \times 10^{-13}$$

A very small fraction!!

1.2.2 Coulomb's Law

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3. A point charge of $+3.00 \times 10^{-6}\,\mathrm{C}$ is $12.0\,\mathrm{cm}$ distant from a second point charge of $-1.50 \times 10^{-6}\,\mathrm{C}$. Calculate the magnitude of the force on each charge. [HRW6 22-2]

Being of opposite signs, the two charges *attract* one another, and the magnitude of this force is given by Coulomb's law (Eq. 1.3),

$$\begin{aligned} F &= k\frac{|q_1 q_2|}{r^2} \\ &= (8.99 \times 10^{9}\,\tfrac{\mathrm{N \cdot m^2}}{\mathrm{C^2}})\frac{(3.00 \times 10^{-6}\,\mathrm{C})(1.50 \times 10^{-6}\,\mathrm{C})}{(12.0 \times 10^{-2}\,\mathrm{m})^2} = 2.81\,\mathrm{N} \end{aligned}$$

Each charge experiences a force of attraction of magnitude 2.81 N.

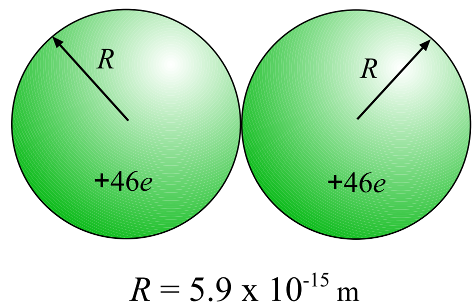

Figure 1.2: Simple picture of a nucleus just after fission. Uniformly charged spheres are "touching".

4. What must be the distance between point charge $q_1 = 26.0\,\mu\mathrm{C}$ and point charge $q_2 = -47.0\,\mu\mathrm{C}$ for the electrostatic force between them to have a magnitude of $5.70\,\mathrm{N}$? [HRW6 22-1]

We are given the charges and the magnitude of the (attractive) force between them. We can use Coulomb's law to solve for r, the distance between the charges:

$$F = k\frac{|q_1 q_2|}{r^2} \qquad \Longrightarrow \qquad r^2 = k\frac{|q_1 q_2|}{F}$$

Plug in the given values:

$$r^2 = (8.99 \times 10^9\,\tfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}})\frac{(26.0 \times 10^{-6}\,\mathrm{C})(47.0 \times 10^{-6}\,\mathrm{C})}{(5.70\,\mathrm{N})} = 1.93\,\mathrm{m}^2$$

This gives:

$$r = \sqrt{1.93\,\mathrm{m}^2} = 1.39\,\mathrm{m}$$

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5. In fission, a nucleus of uranium–238, which contains 92 protons, divides into two smaller spheres, each having 46 protons and a radius of $5.9 \times 10^{-15}\,\mathrm{m}$. What is the magnitude of the repulsive electric force pushing the two spheres apart? [Ser4 23-6]

The basic picture of the nucleus after fission described in this problem is as shown in Fig. 1.2. (Assume that the edges of the spheres are in contact just after the fission.) Now, it is true that Coulomb's law only applies to two *point* masses, but it seems reasonable to take the separation distance r in Coulomb's law to be the distance between the centers of the spheres. (This procedure is *exactly* correct for the gravitational forces between two spherical objects, and because Coulomb's law is another inverse–square force law it turns out to be exactly correct in the latter case as well.)

The charge of each sphere (that is, each nucleus) here is

$$q = +Ze = 46(1.602 \times 10^{-19}\,\mathrm{C}) = 7.369 \times 10^{-18}\,\mathrm{C}\ .$$

The separation of the centers of the spheres is $2R$, so the distance we use in Coulomb's law is

$$r = 2R = 2(5.9 \times 10^{-15}\,\mathrm{m}) = 1.18 \times 10^{-14}\,\mathrm{m}$$

so from Eq. 1.3 the magnitude of the force between the two charged spheres is

$$\begin{aligned} F &= k\frac{|q_1|\,|q_2|}{r^2} \\ &= (8.99 \times 10^9\,\tfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}})\frac{(7.369 \times 10^{-18}\,\mathrm{C})(7.369 \times 10^{-18}\,\mathrm{C})}{(1.18 \times 10^{-14}\,\mathrm{m})^2} = 3.5 \times 10^3\,\mathrm{N}\ . \end{aligned}$$

The force between the two fission fragments has magnitude $3.5 \times 10^3\,\mathrm{N}$, and it is a *repulsive* force since the fragments are both positively charged.

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6. Two small positively charged spheres have a combined charge of $5.0 \times 10^{-5}\,\mathrm{C}$. If each sphere is repelled from the other by an electrostatic force of $1.0\,\mathrm{N}$ when the spheres are $2.0\,\mathrm{m}$ apart, what is the charge on each sphere? [HRW5 22-12]

We are are not given the values of the individual charges; let them be q_1 and q_2. The condition on the combined charge of the spheres gives us:

$$q_1 + q_2 = 5.0 \times 10^{-5}\,\mathrm{C}\ . \tag{1.5}$$

The next condition concerns the electrostatic force, and so it involves Coulomb's Law. Now, Eq. 1.3 involves the *absolute values* of the charges so we need to be careful with the algebra…but in this case we know that both charges are positive because their sum is positive and they *repel* each other. Thus $|q_1| = q_1$ and $|q_2| = q_2$, and the next condition gives us:

$$F = k\frac{q_1 q_2}{r^2} = 1.0\,\mathrm{N}$$

As we know k and r, this give us the value of the product of the charges:

$$q_1 q_2 = \frac{(1.0\,\mathrm{N})r^2}{k} = \frac{(1.0\,\mathrm{N})(2.0\,\mathrm{m})^2}{\left(8.99 \times 10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)} = 4.449 \times 10^{-10}\,\mathrm{C^2} \tag{1.6}$$

With Eqs. 1.5 and 1.6 we have *two* equations for the *two* unknowns q_1 and q_2. We *can* solve for them; the rest is math! Here's my approach to solving the problem:

From Eq. 1.5 we have:

$$q_2 = 5.0 \times 10^{-5}\,\mathrm{C} - q_1 \tag{1.7}$$

Substitute for q_2 in Eq. 1.6 and get:

$$q_1(5.0 \times 10^{-5}\,\mathrm{C} - q_1) = 4.449 \times 10^{-10}\,\mathrm{C^2}$$

which gives us a quadratic equation for q_1:

$$q_1^2 - (5.0 \times 10^{-5}\,\mathrm{C})q_1 + 4.449 \times 10^{-10}\,\mathrm{C^2} = 0$$

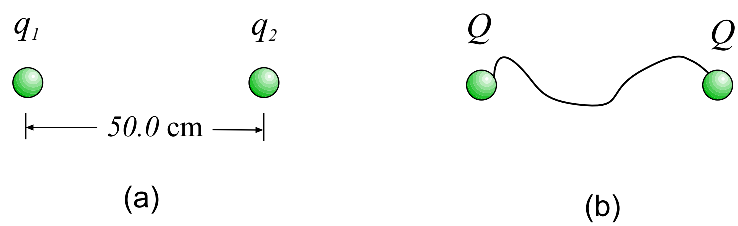

Figure 1.3: (a) Two unknown charges on identical conducting spheres, separated by 50.0 cm, in Example 7. (b) When joined by a wire, the charge evenly divides between the spheres with charge Q on each, such that $q_1 + q_2 = 2Q$.

which we *all* know how to solve. The two possibilities for q_1 are:

$$q_1 = \frac{(5.0 \times 10^{-5}) \pm \sqrt{(5.0 \times 10^{-5})^2 - 4(4.449 \times 10^{-10})}}{2}\,\mathrm{C} = \begin{cases} 3.84 \times 10^{-5}\,\mathrm{C} \\ 1.16 \times 10^{-5}\,\mathrm{C} \end{cases}$$

(Hmm... how do we deal with *two* answers? We'll see...)

Using the two possibilities for q_1 give:

$$q_1 = 3.84 \times 10^{-5}\,\mathrm{C} \quad \Longrightarrow \quad q_2 = 5.0 \times 10^{-5}\,\mathrm{C} - q_1 = 1.16 \times 10^{-5}\,\mathrm{C}$$

$$q_1 = 1.16 \times 10^{-5}\,\mathrm{C} \quad \Longrightarrow \quad q_2 = 5.0 \times 10^{-5}\,\mathrm{C} - q_1 = 3.84 \times 10^{-5}\,\mathrm{C}$$

Actually, these are both the *same* answer, because our numbering of the charges was arbitrary. The answer is that one of the charges is 1.16×10^{-5} C and the other is 3.84×10^{-5} C.

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7. Two identical conducting spheres, fixed in place, attract each other with an electrostatic force of 0.108 N when separated by 50.0 cm, center-to-center. The spheres are then connected by a thin conducting wire. When the wire is removed, the spheres repel each other with an electrostatic force of 0.360 N. What were the initial charges on the spheres? [HRW6 22-7]

The initial configuration of the spheres is shown in Fig. 1.3(a). Let the charges on the spheres be q_1 and q_2. If the force of attraction between them has magnitude 0.108 N, then Coulomb's law gives us

$$F = k\frac{|q_1 q_2|}{r^2} = (8.99 \times 10^9\,\tfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}})\frac{|q_1 q_2|}{(0.500\,\mathrm{m})^2} = 0.108\,\mathrm{N}$$

from which we get

$$|q_1 q_2| = \frac{(0.108\,\mathrm{N})(0.500\,\mathrm{m})^2}{(8.99 \times 10^9\,\tfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}})} = 3.00 \times 10^{-12}\,\mathrm{C^2}$$

But since we are told that the charges *attract* one another, we know that q_1 and q_2 have opposite signs and so their product must be *neagtive*. So we can drop the absolute value sign if we write

$$q_1 q_2 = -3.00 \times 10^{-12}\,\mathrm{C^2} \tag{1.8}$$

Then the two spheres are joined by a wire. The charge is now free to re–distribute itself between the two spheres and since they are identical the total excess charge (that is, $1_1 + q_2$) will be *evenly* divided between the two spheres. If the new charge on each sphere is Q, then

$$Q + Q = 2Q = q_1 + q_2 \tag{1.9}$$

The force of repulsion between the spheres is now 0.0360 N, so that

$$F = k\frac{Q^2}{r^2} = (8.99 \times 10^9\ \tfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}})\frac{Q^2}{(0.500\,\mathrm{m})^2} = 0.0360\,\mathrm{N}$$

which gives

$$Q^2 = \frac{(0.0360\,\mathrm{N})(0.500\,\mathrm{m})^2}{(8.99 \times 10^9\ \tfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}})} = 1.00 \times 10^{-12} C^2$$

We don't know what the sign of Q is, so we can only say:

$$Q = \pm 1.00 \times 10^{-6}\,\mathrm{C} \tag{1.10}$$

Putting 1.10 into 1.9, we get

$$q_1 + q_2 = 2Q = \pm 2.00 \times 10^{-6}\,\mathrm{C} \tag{1.11}$$

and now 1.8 and 1.11 give us two equations for the two unknowns q_1 and q_2, and we're in business!

First, choosing the + sign in 1.11 we have

$$q_2 = 2.00 \times 10^{-6}\,\mathrm{C} - q_1 \tag{1.12}$$

and substituting this into 1.8 we have:

$$q_1(2.00 \times 10^{-6}\,\mathrm{C} - q_1) = -3.00 \times 10^{-12}\,\mathrm{C}^2$$

which we can rewrite as

$$q_1^2 - (2.00 \times 10^{-6}\,\mathrm{C})q_1 - 3.00 \times 10^{-12}\,\mathrm{C}^2 = 0$$

which is a quadratic equation for q_1. When we find the solutions; we get:

$$q_1 = 3.00 \times 10^{-6}\,\mathrm{C} \qquad \text{or} \qquad q_1 = -1.00 \times 10^{-6}\,\mathrm{C}$$

Putting these possibilities into 1.12 we find

$$q_2 = -1.00 \times 10^{-6}\,\mathrm{C} \qquad \text{or} \qquad q_2 = 3.00 \times 10^{-6}\,\mathrm{C}$$

but these really give the same answer: One charge is $-1.00 \times 10^{-6}\,\mathrm{C}$ and the other is $+3.00 \times 10^{-6}\,\mathrm{C}$.

Now make the other choice in 1.11. Then we have

$$q_2 = -2.00 \times 10^{-6}\,\mathrm{C} - q_1 \tag{1.13}$$

Putting this into 1.8 we have:

$$q_1(-2.00 \times 10^{-6}\,\mathrm{C} - q_1) = -3.00 \times 10^{-12}\,\mathrm{C}^2$$

which we can rewrite as

$$q_1^2 + (2.00 \times 10^{-6}\,\mathrm{C})q_1 - 3.00 \times 10^{-12}\,\mathrm{C}^2 = 0$$

which is a different quadratic equation for q_1, and which has the solutions

$$q_1 = -3.00 \times 10^{-6}\,\mathrm{C} \qquad \text{or} \qquad q_1 = 1.00 \times 10^{-6}\,\mathrm{C}$$

Putting these into 1.13 we get

$$q_2 = 1.00 \times 10^{-6}\,\mathrm{C} \qquad \text{or} \qquad q_2 = -3.00 \times 10^{-6}\,\mathrm{C}$$

but these really give the same answer: One charge is $+1.00 \times 10^{-6}\,\mathrm{C}$ and the other is $-3.00 \times 10^{-6}\,\mathrm{C}$.

So in the end we have two distinct possibilities for the initial charges q_1 and q_2 on the spheres. They are

$$-1.00\,\mu\mathrm{C} \qquad \text{and} \qquad +3.00\,\mu\mathrm{C}$$

and

$$+1.00\,\mu\mathrm{C} \qquad \text{and} \qquad -3.00\,\mu\mathrm{C}$$

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8. A certain charge Q is divided into two parts q and $Q - q$, which are then separated by a certain distance. What must q be in terms of Q to maximize the electrostatic repulsion between the two charges? [HRW6 22-13]

If the distance between the two (new) charges is r, then the magnitude of the force between them is

$$F = k\frac{(Q-q)q}{r^2} = \frac{k}{r^2}(qQ - q^2)\ .$$

(We know that Q and $Q - q$ both have the same sign so that $Q(Q - q)$ is necessarily a positive number. Force between the charges is repulsive.) To find the value of q which give maximum F, take the derivative of F with respect to q and find where it is zero:

$$\frac{dF}{dq} = \frac{k}{r^2}(Q - 2q) = 0$$

which has the solution

$$(Q - 2q) = 0 \qquad \Longrightarrow \qquad q = \frac{Q}{2}\ .$$

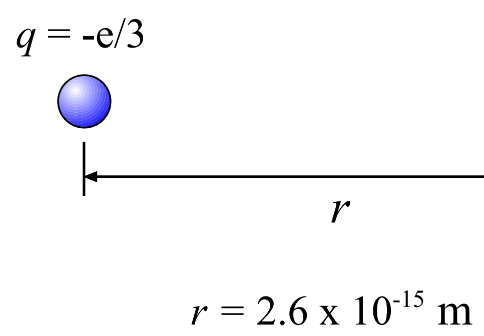

Figure 1.4: Two down quarks, each with charge $-e/3$, separated by 2.6×10^{-15} m, in Example 9.

So the maximum repulsive force is gotten by dividing the original charge Q in half.

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9. A neutron consists of one "up" quark of charge $+\frac{2e}{3}$ and two "down" quarks each having charge $-\frac{e}{3}$. If the down quarks are 2.6×10^{-15} m apart inside the neutron, what is the magnitude of the electrostatic force between them? [HRW5 22-24]

We picture the two down quarks as in Fig. 1.4. We use Coulomb's law to find the force between them. (It is *repulsive* since the quarks have the same charge.) The two charges are:

$$q_1 = q_2 = -\frac{e}{3} = -\frac{(1.60 \times 10^{-19}\,\mathrm{C})}{3} = -5.33 \times 10^{-20}\,\mathrm{C}$$

and the separation is $r = 2.6 \times 10^{-15}$ m. The magnitude of the force is

$$F = k\frac{|q_1|\,|q_2|}{r^2} = \left(8.99 \times 10^9\,\tfrac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)\frac{(5.33 \times 10^{-20}\,\mathrm{C})(5.33 \times 10^{-20}\,\mathrm{C})}{(2.6 \times 10^{-15}\,\mathrm{m})^2} = 3.8\,\mathrm{N}$$

The magnitude of the (repulsive) force is 3.8 N.

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10. The charges and coordinates of two charged particles held fixed in the xy plane are: $q_1 = +3.0\,\mu\mathrm{C}$, $x_1 = 3.5\,\mathrm{cm}$, $y_1 = 0.50\,\mathrm{cm}$, and $q_2 = -4.0\,\mu\mathrm{C}$, $x_2 = -2.0\,\mathrm{cm}$, $y_2 = 1.5\,\mathrm{cm}$. (a) Find the magnitude and direction of the electrostatic force on q_2. (b) Where could you locate a third charge $q_3 = +4.0\,\mu\mathrm{C}$ such that the net electrostatic force on q_2 is zero? [HRW6 22-12]

(a) First, make a sketch giving the locations of the charges. This is done in Fig. 1.5. (Clearly, q_2 will be *attracted* to q_1; the force on it will be to the right and downward.)

Find the distance between q_2 and q_1. It is

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2.0 - 3.5)^2 + (1.5 - 0.50)^2}\,\mathrm{cm} = 5.59\,\mathrm{cm} \end{aligned}$$

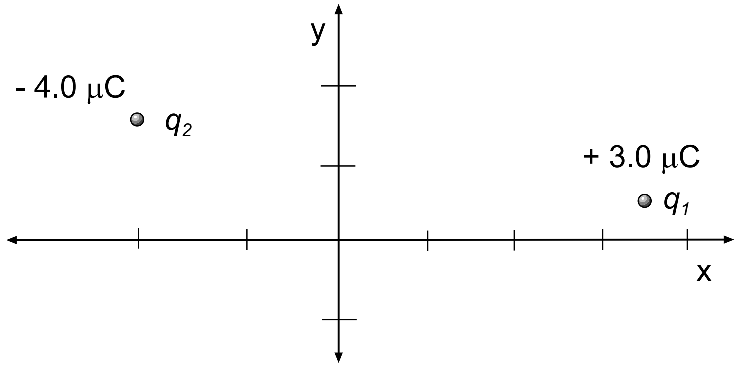

Figure 1.5: Locations of charges in Example 10.

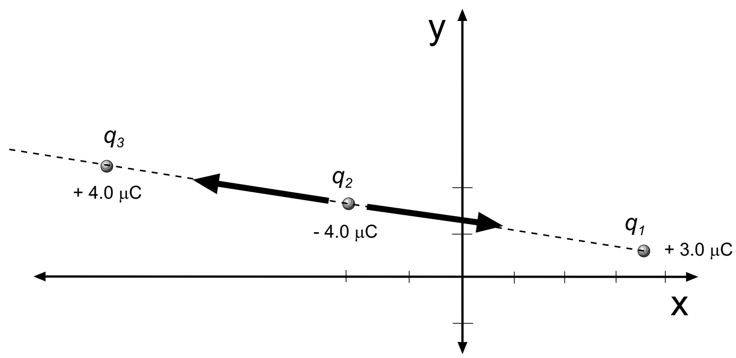

Figure 1.6: Placement of q_3 such as to give zero net force on q_2.

Then by Coulomb's law the force on q_2 has magnitude

$$F = k\frac{|q_1||q_2|}{r^2} = (8.99 \times 10^9\ \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}})\frac{(3.0 \times 10^{-6}\,\mathrm{C})(4.0 \times 10^{-6}\,\mathrm{C})}{(5.59 \times 10^{-2}\,\mathrm{m})^2} = 35\,\mathrm{N}$$

Since q_2 is attracted to q_1, the *direction* of this force is the same as the vector which points from q_2 to q_1. That vector is

$$\mathbf{r}_{12} = (x_1 - x_2)\mathbf{i} + (y_1 - y_2)\mathbf{j} = (5.5\,\mathrm{cm})\mathbf{i} + (-1.0\,\mathrm{cm})\mathbf{j}$$

The direction (angle) of this vector is

$$\theta = \tan^{-1}\left(\frac{-1.0}{5.5}\right) = -10.3^\circ$$

(b) The force which the $+4.0\,\mu\mathrm{C}$ charge exerts on q_2 must cancel the force we calculated in part (a) (i.e. the attractive force from q_1). Since this charge will exert an attractive force on q_2, we must place it on the line which joins q_1 and q_2 but on the *other* side of q_2. This is shown in Fig. 1.6.

First, find the distance r' between q_3 and q_2. The force of q_3 on q_2 must also have magnitude 35 N; this allows us to solve for r':

$$F = k\frac{|q_2||q_3|}{r'^2} \qquad \Rightarrow \qquad r'^2 = k\frac{|q_2||q_3|}{F}$$

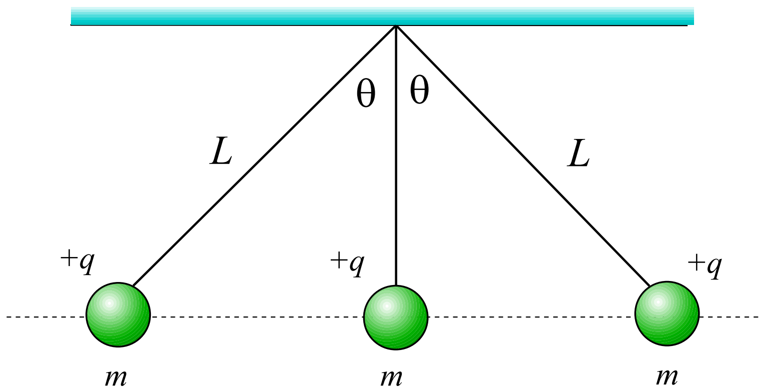

Figure 1.7: Charged masses hang from strings, as described in Example 11.

Plug in the numbers:

$$r'^2 = (8.99 \times 10^9 \, \tfrac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}) \frac{(4.0 \times 10^{-6}\,\mathrm{C})(4.0 \times 10^{-6}\,\mathrm{C})}{(35\,\mathrm{N})} = 4.1 \times 10^{-3}\,\mathrm{m}$$

$$r' = 6.4 \times 10^{-2}\,\mathrm{m} = 6.4\,\mathrm{cm}$$

This is the distance q_3 from q_2; we also know that being opposite q_1, its direction is

$$\theta' = 180^\circ - 10.3^\circ = 169.7^\circ$$

from q_2. So the *displacement* of q_3 from q_2 is given by:

$$\Delta x = r' \cos\theta' = (6.45\,\mathrm{cm}) \cos 169.7^\circ = -6.35\,\mathrm{cm}$$

$$\Delta y = r' \sin\theta' = (6.45\,\mathrm{cm}) \sin 169.7^\circ = +1.15\,\mathrm{cm}$$

Adding these differences to the coordinates of q_2 we find:

$$x_3 = x_2 + \Delta x = -2.0\,\mathrm{cm} - 6.35\,\mathrm{cm} = -8.35\,\mathrm{cm}$$

$$y_3 = y_2 + \Delta y = +1.5\,\mathrm{cm} + 1.15\,\mathrm{cm} == 2.65\,\mathrm{cm}$$

The charge q_3 should be placed at the point $(-8.35\,\mathrm{cm}, 2.65\,\mathrm{cm})$.

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11. Three identical point charges, each of mass $m = 0.100\,\mathrm{kg}$ and charge q hang from three strings, as in Fig. 1.7. If the lengths of the left and right strings are $L = 30.0\,\mathrm{cm}$ and angle $\theta = 45.0^\circ$, determine the value of q. [Ser4 23-10]

Make a free–body diagram in order to understand things! Choose the leftmost mass in Fig. 1.7. The forces on this mass are shown in Fig. 1.8. Gravity pulls down with a force mg; the string tension pulls as shown with a force of magnitude T. Both of the other charged masses exert forces of electrostatic repulsion on this mass. The charge in the middle exerts

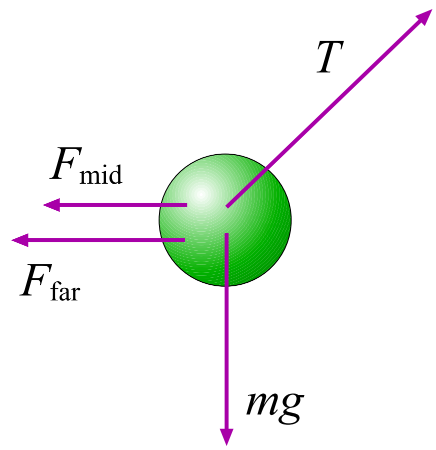

Figure 1.8: Forces acting on the leftmost charged mass in Example 11.

a force of magnitude $F_{\rm mid}$; the rightmost (far) charge exerts a force of magnitude $F_{\rm far}$. Both forces are directed to the left.

We can get expressions for $F_{\rm mid}$ and $F_{\rm far}$ using Coulomb's law. The distance between the left charge and the middle charge is

$$r_1 = (30.0\,\mathrm{cm})\sin 45.0^\circ = 21.2\,\mathrm{cm} = 0.212\,\mathrm{m}$$

and since both charges are $+q$ we have

$$F_{\rm mid} = k\frac{q^2}{(0.212\,\mathrm{m})^2} \ .$$

Likewise, the distance between the left charge and the rightmost charge is

$$r_2 = 2(30.0\,\mathrm{cm})\sin 45.0^\circ = 2(0.212\,\mathrm{m}) = 0.424\,\mathrm{m}$$

so that we have

$$F_{\rm far} = k\frac{q^2}{(0.424\,\mathrm{m})^2} \ .$$

The vertical forces on the mass must sum to zero. This gives us:

$$T\sin 45.0^\circ - mg = 0 \qquad \Longrightarrow \qquad T = \frac{mg}{\sin 45.0^\circ} = 1.39\,\mathrm{N}$$

where we have used the given value of m to evaluate T.

The horizontal forces must also sum to zero, and this gives us:

$$-F_{\rm mid} - F_{\rm far} + T\cos 45.0^\circ = 0$$

Substitute for $F_{\rm mid}$ and $F_{\rm far}$ and get:

$$-k\frac{q^2}{(0.212\,\mathrm{m})^2} - k\frac{q^2}{(0.424\,\mathrm{m})^2} + T\cos 45.0^\circ = 0 \tag{1.14}$$

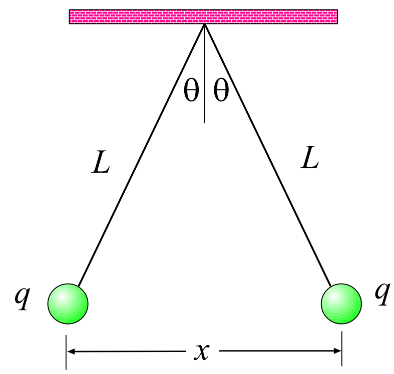

Figure 1.9: Charged masses hang from strings, as described in Example 12.

Since we have already found T, the only unknown in this equation is q. The *physics* part of the problem is done!

A little rearranging of Eq. 1.14 gives us:

$$kq^2\left(\frac{1}{(0.212\,\mathrm{m})^2} + \frac{1}{(0.424\,\mathrm{m})^2}\right) = T\cos 45.0^\circ$$

The sum in the big parenthesis is equal to $27.8\,\mathrm{m}^{-2}$ and with this we can solve for q:

$$\begin{aligned} q^2 &= \frac{T\cos 45.0^\circ}{k(27.8\,\mathrm{m}^{-2})} \\ &= \frac{(1.39\,\mathrm{N})\cos 45.0^\circ}{\left(8.99\times 10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)(27.8\,\mathrm{m}^{-2})} = 3.93\times 10^{-12}\,\mathrm{C}^2 \end{aligned}$$

And then:

$$q = 1.98\times 10^{-6}\,\mathrm{C} = 1.98\,\mu\mathrm{C}$$

---

12. In Fig. 1.9, two tiny conducting balls of identical mass and identical charge q hang from nonconducting threads of length L. Assume that θ is so small that $\tan\theta$ can be replaced by its approximate equal, $\sin\theta$. (a) Show that for equilibrium,

$$x = \left(\frac{q^2 L}{2\pi\epsilon_0 mg}\right)^{1/3} ,$$

where x is the separation between the balls. (b) If $L = 120\,\mathrm{cm}$, $m = 10\,\mathrm{g}$ and $x = 5.0\,\mathrm{cm}$, what is q? [HRW6 22-15]

(a) We draw a free-body diagram for one of the charge (say, the left one). This is done in Fig. 1.10. The forces acting on the charged ball are the string tension T, the downward force of gravity mg and the force of electrostatic repulsion from the other charged ball, F_{elec}. The direction of this for is to the left because the other ball, having the same charge exerts a

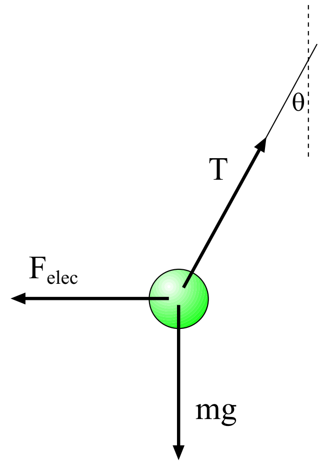

Figure 1.10: The forces acting on one of the charged masses in Example 12.

repulsive force which must point horizontally to the left because of the symmetric position of the other ball.

We do know the magnitude of the force of electrostatic repulsion; from Coulomb's law it is

$$F_{\mathrm{elec}} = k\frac{q^2}{x^2}$$

The ball is in static equilibrium, so the forces on the ball sum to zero. The vertical components add to zero, which gives us:

$$T\cos\theta = mg$$

and from the horizontal components we get

$$T\sin\theta = F_{\mathrm{elec}} = k\frac{q^2}{x^2}$$

Divide the second of these equations by the first one and get:

$$\frac{T\sin\theta}{T\cos\theta} = \tan\theta = \frac{kq^2}{mgx^2} \tag{1.15}$$

Now the problem says that the angle θ is so small that we can safely replace $\tan\theta$ by $\sin\theta$ (they are nearly the same for "small" angles). But from the geometry of the problem we can express $\sin\theta$ as:

$$\sin\theta = \frac{x/2}{L} = \frac{x}{2L}$$

Using all of this in Eq. 1.15 we get:

$$\frac{x}{2L} \approx \frac{kq^2}{mgx^2}$$

Now we can solve for x because a little algebra gives:

$$x^3 = \frac{kq^2(2L)}{mg} = \frac{2q^2L}{4\pi\epsilon_0 mg} = \frac{q^2L}{2\pi\epsilon_0 mg} \tag{1.16}$$

which then gives the answer for x,

$$x = \left(\frac{q^2L}{2\pi\epsilon_0 mg}\right)^{1/3}$$

(b) Rearranging Eq. 1.16 we find:

$$q^2 = \frac{2\pi\epsilon_0 mgx^3}{L}$$

and plugging in the given values (in SI units, of course), we get:

$$q^2 = \frac{2\pi(8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\cdot m}^2})(10 \times 10^{-3}\,\mathrm{kg})(9.80\,\frac{\mathrm{m}}{\mathrm{s}^2})}{(1.20\,\mathrm{m})} = 5.68 \times 10^{-16}\,\mathrm{C}^2$$

and then we find q (note the ambiguity in sign!):

$$q = \pm 2.4 \times 10^{-8}\,\mathrm{C}\ \ .$$