Chapter 6

Waves II: Sound Waves

6.1 The Important Stuff

6.1.1 Sound Waves; The Speed of Sound

A sound wave is a longitudinal wave in an elastic medium (which could be a gas, liquid or solid). In such a wave the particles of the medium oscillate back and forth *along the direction in which the wave travels* such that regions of high and low density are created. It is these regions of compression and rarefaction which make up the wave fronts which travel through space and carry energy.

The waves have a speed which comes from the elastic properties of the medium.

6.1.2 Interference of Sound Waves

6.1.3 Intensity and Sound Level

$$I = \tfrac{1}{2}\rho v \omega^2 s_m^2 \tag{6.1}$$

$$I = \frac{P}{4\pi r^2} \tag{6.2}$$

$$\beta = (10\,\mathrm{dB})\log_{10}\frac{I}{I_0} \tag{6.3}$$

6.1.4 Standing Waves in Pipes

6.1.5 Beats

$$f_{\mathrm{beat}} = |f_1 - f_2| \tag{6.4}$$

6.1.6 The Doppler Effect

When a source or sound is in motion — or if an "observer" is in motion — or if *both* are in motion — the observer will detect a different frequency from what the source is producing.

If a source of sound moves toward an observer the wavefronts are bunched up closer together than they would be otherwise (though they travel the same speed). This results in a greater frequency at which they strike the observer. If v_S is the speed of the source toward or away from the observer, v is the speed of sound, and f is the frequency at which the source is making the sound, then the observer hears a frequency f' given by:

$$f' = f\left(\frac{v}{v \mp v_S}\right) \tag{6.5}$$

Here the top sign goes with motion of the source *toward* the observer, the bottom sign with motion *away*.

If the source of sound is stationary and the observer is moving toward the source or away from it, he will also hear a different frequency, but for a different reason. Here the observer encounters the wave fronts which have their usual separation distance (i.e. wavelength) but because of his motion, they are (effectively) coming at him with a different speed, and as a result he receives them at a different frequency. If v_O is the speed of the source toward or away from the observer, v is the speed of sound, and f is the frequency at which the source is making the sound, then the observer hears a frequency f' given by:

$$f' = f\left(\frac{v \pm v_O}{v}\right) \tag{6.6}$$

Again, the top sign goes with motion "toward" and the bottom sign with motion "away".

If both the source and obeserver are in motion then in fact Eqs. **??** and **??** combine as

$$f' = f\left(\frac{v \pm v_O}{v \mp v_S}\right) = \left(\frac{1 \pm \frac{v_O}{v}}{1 \mp \frac{v_S}{v}}\right) \tag{6.7}$$

6.2 Worked Examples

Unless otherwise specified, we will take the speed of sound in air to be $343\,\frac{\mathrm{m}}{\mathrm{s}}$ and the (mass) density of air as $1.21\,\frac{\mathrm{kg}}{\mathrm{m}^3}$.

6.2.1 Sound Waves; The Speed of Sound

1. A stone is dropped into a well. The sound of the splash is heard $3.00\,\mathrm{s}$ later. What is the depth of the well? [HRW6 18-7]

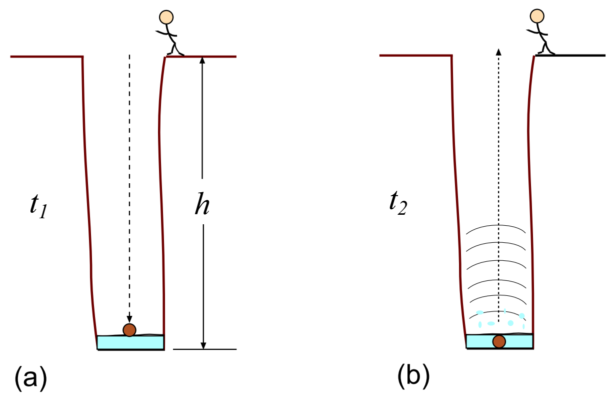

Figure 6.1: (a) Rock falls to bottom of well, in a time t_1. (b) Sound of splash travels back up the well, in a time t_2.

There are two time intervals we need to think about: The time t_1 it takes for the rock to fall to the bottom of the well and the time t_2 it it takes for the sound to travel back up the well to the listener, as diagrammed in Fig. 6.1. . The *sum* of these two times is 3.00 s:

$$t_1 + t_2 = 3.00\,\mathrm{s} \tag{6.8}$$

If the depth of the well is h, then from our ample experience in the kinematics of freely–falling objects we can relate t_1 to h. We have:

$$\tfrac{1}{2}gt_1^2 = \tfrac{1}{2}(9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2})t_1^2 = h \tag{6.9}$$

Now, unlike a falling rock, sound travels at a *constant* speed (which we'll assume is $v_{\mathrm{sound}} = 343\,\frac{\mathrm{m}}{\mathrm{s}}$) so that the relation between t_2 and h is

$$v_{\mathrm{sound}}t_2 = (343\,\tfrac{\mathrm{m}}{\mathrm{s}})t_2 = h \tag{6.10}$$

We can combine 6.9 and 6.10 (eliminate h) to get:

$$\tfrac{1}{2}(9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2})t_1^2 = (343\,\tfrac{\mathrm{m}}{\mathrm{s}})t_2$$

Since 6.8 gives us $t_2 = 3.00\,\mathrm{s} - t_1$, we can substitute for t_2 to get an equation in *one* unknown:

$$\tfrac{1}{2}(9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2})t_1^2 = (343\,\tfrac{\mathrm{m}}{\mathrm{s}})(3.00\,\mathrm{s} - t_1)$$

Dropping the units for simplicity, we can rearrange this to:

$$(4.9)t_1^2 + (343)t_1 - 1029 = 0$$

Use the quadratic formula to find t_1 (just get the positive root; the negative one is meaningless here):

$$t_1 = \frac{-343 + \sqrt{(343)^2 + 4(4.9)(1029)}}{2(4.9)} = 2.88\,\mathrm{s}$$

so that we can get t_2:

$$t_2 = 3.00\,\mathrm{s} - 2.88\,\mathrm{s} = 0.12\,\mathrm{s}$$

and then 6.10 gives h;

$$h = (343\,\tfrac{\mathrm{m}}{\mathrm{s}})(0.12\,\mathrm{s}) = 40.7\,\mathrm{m}$$

The depth of the well is 40.7 m.

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2. The audible frequency range for normal hearing is from about 20 Hz to 20 kHz. What are the wavelengths of sound waves at these frequencies? [HRW6 18-8]

The wavelength and frequency of a wave are related by $\lambda f = v$, where v is the speed of the wave. Here we take $v = 343\,\frac{\mathrm{m}}{\mathrm{s}}$; for a sound wave of frequency 20 Hz the wavelength is

$$\lambda = \frac{v}{f} = \frac{(343\,\frac{\mathrm{m}}{\mathrm{s}})}{(20.0\,\mathrm{Hz})} = \frac{(343\,\frac{\mathrm{m}}{\mathrm{s}})}{(20.0\,\mathrm{s}^{-1})} = 17.2\,\mathrm{m}$$

For the wave of frequency 20 kHz, the wavelength is

$$\lambda = \frac{v}{f} = \frac{(343\,\frac{\mathrm{m}}{\mathrm{s}})}{(20.0\,\mathrm{kHz})} = \frac{(343\,\frac{\mathrm{m}}{\mathrm{s}})}{(20.0 \times 10^3\,\mathrm{s}^{-1})} = 1.72 \times 10^{-2}\,\mathrm{m} = 1.72\,\mathrm{cm}$$

6.2.2 Intensity and Sound Level

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3. A source emits sound waves isotropically. The intensity of the waves 2.50 m from the source is $1.91 \times 10^{-4}\,\mathrm{W/m^2}$. Assuming that the energy of the waves is conserved, find the power of the source. [HRW6 18-17]

Eq. 6.2 gives the relation between power, intensity and distance for an isotropic (point) source of sound waves: $I = P/(4\pi r^2)$. We are given the intensity I at a given distance r, so the power of the source is

$$\begin{aligned} P &= 4\pi r^2 I = 4\pi(2.50\,\mathrm{m})^2(1.91 \times 10^{-4}\,\tfrac{\mathrm{W}}{\mathrm{m}^2}) \\ &= 1.50 \times 10^{-2}\,\mathrm{W} = 15.0\,\mathrm{mW} \end{aligned}$$

The power of the source is 15.0 mW.

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4. A sound wave of frequency 300 Hz has an intensity of $1.00\,\mu\mathrm{W/m^2}$. What is the amplitude of the air oscillations caused by this wave? [HRW6 18-19]

Eq. 6.1 gives the intensity of a sound wave in terms of the amplitude of the air oscillations (s_m) and other things. The formula requires the mass density of air ($1.21\,\frac{\mathrm{kg}}{\mathrm{m}^3}$), the speed of sound ($343\,\frac{\mathrm{m}}{\mathrm{s}}$) and the angular frequency of oscillation of the air, namely

$$\omega = 2\pi f = 2\pi(300\,\mathrm{Hz}) = 1.88 \times 10^3\,\mathrm{s}^{-1}$$

Then we find:

$$
\begin{aligned}
s_m^2 &= \frac{2I}{\rho v \omega^2} \\
&= \frac{2(1.00\times 10^{-6}\,\frac{\mathrm{W}}{\mathrm{m}^2})}{(1.21\,\frac{\mathrm{kg}}{\mathrm{m}^3})(343\,\frac{\mathrm{m}}{\mathrm{s}})(1.88\times 10^3\,\mathrm{s}^{-1})} = 1.36\times 10^{-15}\,\mathrm{m}^2
\end{aligned}
$$

So then the amplitude of the air oscillations is

$$
s_m = 3.7\times 10^{-8}\,\mathrm{m} = 37\,\mathrm{nm} \quad .
$$

This is a *very* small length, but it should be remembered that it represents an *average* in the shifts of the positions of the air molecules.

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5. Two sounds differ in sound level by $1.00\,\mathrm{dB}$. What is the ratio of the greater intensity to the smaller intensity? [HRW6 18-20]

Let β_2 be the sound level of the louder sound and I_2 its intensity. Likewise, β_1 and I_1 are the sound level and intensity of the weaker sound. We are given that $\beta_2 - \beta_1 = 1.00$. Use the definition of the sound level to get some information on the intensities:

$$
\begin{aligned}
\beta_2 - \beta_1 &= 1.00 \\
&= (10\,\mathrm{dB})\log_{10}\left(\frac{I_2}{I_0}\right) - (10\,\mathrm{dB})\log_{10}\left(\frac{I_1}{I_0}\right) \\
&= (10\,\mathrm{dB})\left[\log_{10}\left(\frac{I_2}{I_0}\right) - \log_{10}\left(\frac{I_1}{I_0}\right)\right] \qquad (6.11)
\end{aligned}
$$

(In the last step, we factored out $(10\,\mathrm{dB})$ from each term.) Now we observe the the logs in the last step can combine (a minus sign *inverts* the argument of a logarithm):

$$
\begin{aligned}
\log_{10}\left(\frac{I_2}{I_0}\right) - \log_{10}\left(\frac{I_1}{I_0}\right) &= \log_{10}\left(\frac{I_2}{I_0}\right) + \log_{10}\left(\frac{I_0}{I_1}\right) \\
&= \log_{10}\left(\frac{I_2}{I_0}\cdot\frac{I_0}{I_1}\right) \\
&= \log_{10}\left(\frac{I_2}{I_1}\right)
\end{aligned}
$$

Put this into 6.11 and find:

$$
1.00\,\mathrm{dB} = (10\,\mathrm{dB})\log_{10}\left(\frac{I_2}{I_1}\right)
$$

This gives us:

$$
\log_{10}\left(\frac{I_2}{I_1}\right) = \frac{1.00}{10} = 0.100
$$

and raising 10 to the power of both sides gives:

$$\frac{I_2}{I_1} = 10^{0.100} = 1.26$$

The *ratio* of the two sound intensities is 1.26.

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6. A point source emits $30.0\,\mathrm{W}$ of sound isotropically. A small microphone intercepts the sound in an area of $0.7500\,\mathrm{cm}^2$, $200\,\mathrm{m}$ from the source. Calculate (a) the sound intensity there and (b) the power intercepted by the microphone. [HRW6 18-28]

(a) Eq. 6.2 gives the intensity of an isotropic sound wave from the power of the source and the distance:

$$I = \frac{P}{4\pi r^2} = \frac{(30.0\,\mathrm{W})}{4\pi(200\,\mathrm{m})^2} = 5.97 \times 10^{-5}\,\frac{\mathrm{W}}{\mathrm{m}^2}$$

(b) The power intercepted by the microphone is the intensity at its location multiplied by the area of its sound–sensitive element (assuming it is held face–on to the direction of the incoming wave): $P_{\mathrm{micr}} = IA_{\mathrm{micr}}$. Using $1\,\mathrm{m}^2 = 10^{-4}\,\mathrm{cm}^2$, this gives:

$$P_{\mathrm{micr}} = IA_{\mathrm{micr}} = (5.97 \times 10^{-5}\,\tfrac{\mathrm{W}}{\mathrm{m}^2})(0.750 \times 10^{-4}\,\mathrm{m}^2) = 4.48 \times 10^{-9}\,\mathrm{W}\ .$$

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7. A car horn emits a $380\,\mathrm{Hz}$ sound. If the car moves at $17\,\frac{\mathrm{m}}{\mathrm{s}}$ with its horn blasting, what frequency will a person standing in front of the car hear? [Wolf 14-42]

Here the source is in motion so that eq. **??** is applicable. Use the top sign for motion "toward"; using $v = 343\,\frac{\mathrm{m}}{\mathrm{s}}$ as the speed of sound, the listener hears a frequency

$$f' = f\left(\frac{v}{v - v_S}\right) = (380\,\mathrm{Hz})\left(\frac{343\,\frac{\mathrm{m}}{\mathrm{s}}}{343 - 17\,\frac{\mathrm{m}}{\mathrm{s}}}\right) = (380\,\mathrm{Hz})(1.05) = 400\,\mathrm{Hz}$$

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8. A fire truck's siren at rest wails at $1400\,\mathrm{Hz}$; standing by the roadside as the truck approaches, you hear it at $1600\,\mathrm{Hz}$. How fast is the truck going? [Wolf 14-44]

Again use the formula for the frequency heard when the source is moving; this timne v_S is unknown. Use $343\,\frac{\mathrm{m}}{\mathrm{s}}$ for the speed of sound:

$$f' = f\left(\frac{v}{v - v_S}\right) \qquad \Longrightarrow \qquad v - v_S = \frac{f}{f'}v = \left(\frac{1400}{1600}\right)(343\,\tfrac{\mathrm{m}}{\mathrm{s}}) = 300\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

Rearrange and get v_S:

$$v_S = v - 300\,\tfrac{\mathrm{m}}{\mathrm{s}} = 343\,\tfrac{\mathrm{m}}{\mathrm{s}} - 300\,\tfrac{\mathrm{m}}{\mathrm{s}} = 43\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

Appendix A: Conversion Factors

Length	cm	meter	km	in	ft	mi
1 cm =	1	10^{-2}	10^{-5}	0.3937	3.281×10^{-2}	6.214×10^{-6}
1 m =	100	1	10^{-3}	39.37	3.281	6.214×10^{-4}
1 km =	10^5	1000	1	3.937×10^4	3281	06214
1 in =	2.540	2.540×10^{-2}	2.540×10^{-5}	1	8.333×10^{-2}	1.578×10^{-5}
1 ft =	30.48	0.3048	3.048×10^{-4}	12	1	1.894×10^{-4}
1 mi =	1.609×10^5	1609	1.609	6.336×10^4	5280	1

Mass	g	kg	slug	u
1 g =	1	0.001	6.852×10^{-2}	6.022×10^{26}
1 kg =	1000	1	6.852×10^{-5}	6.022×10^{23}
1 slug =	1.459×10^4	14.59	1	8.786×10^{27}
1 u =	1.661×10^{-24}	1.661×10^{-27}	1.138×10^{-28}	1

An object with a *weight* of 1 lb has a *mass* of 0.4536 kg.