Chapter 7

Linear Momentum and Collisions

7.1 The Important Stuff

7.1.1 Linear Momentum

The **linear momentum** of a particle with mass m moving with velocity $\mathbf{v}$ is defined as

$$\mathbf{p} = m\mathbf{v} \tag{7.1}$$

Linear momentum is a *vector*. When giving the linear momentum of a particle you *must* specify its *magnitude* and *direction*. We can see from the definition that its units must be $\frac{\text{kg}\cdot\text{m}}{\text{s}}$. Oddly enough, this combination of SI units does not have a commonly–used named so we leave it as $\frac{\text{kg}\cdot\text{m}}{\text{s}}$!

The momentum of a particle is related to the net force on that particle in a simple way; since the mass of a particle remains constant, if we take the time derivative of a particle's momentum we find

$$\frac{d\mathbf{p}}{dt} = m\frac{d\mathbf{v}}{dt} = m\mathbf{a} = \mathbf{F}_{\text{net}}$$

so that

$$\mathbf{F}_{\text{net}} = \frac{d\mathbf{p}}{dt} \tag{7.2}$$

7.1.2 Impulse, Average Force

When a particle moves freely then interacts with another system for a (brief) period and then moves freely again, it has a definite change in momentum; we define this change as the **impulse I** of the interaction forces:

$$\mathbf{I} = \mathbf{p}_f - \mathbf{p}_i = \Delta\mathbf{p}$$

Impulse is a vector and has the same units as momentum.

When we integrate Eq. 7.2 we can show:

$$\mathbf{I} = \int_{t_i}^{t_f} \mathbf{F}\, dt = \Delta\mathbf{p}$$

We can now define the **average force** which acts on a particle during a time interval Δt. It is:

$$\overline{\mathbf{F}} = \frac{\Delta p}{\Delta t} = \frac{\mathbf{I}}{\Delta t}$$

The value of the average force depends on the time interval chosen.

7.1.3 Conservation of Linear Momentum

Linear momentum is a useful quantity for cases where we have a few particles (objects) which interact with *each other* but not with the rest of the world. Such a system is called an **isolated system**.

We often have reason to study systems where a few particles interact with each other very briefly, with forces that are strong compared to the other forces in the world that they may experience. In those situations, and for that brief period of time, we can treat the particles as if they *were* isolated.

We can show that when two particles interact *only* with each other (i.e. they are isolated) then their total momentum remains constant:

$$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f} \tag{7.3}$$

or, in terms of the masses and velocities,

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f} \tag{7.4}$$

Or, abbreviating $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}$ (total momentum), this is: $\mathbf{P}_i = \mathbf{P}_f$.

It is important to understand that Eq. 7.3 is a *vector* equation; it tells us that the total x component of the momentum is conserved, *and* the total y component of the momentum is conserved.

7.1.4 Collisions

When we talk about a **collision** in physics (between two particles, say) we mean that two particles are moving freely through space until they get close to one another; then, for a short period of time they exert strong forces on each other until they move apart and are again moving freely.

For such an event, the two particles have well-defined momenta $\mathbf{p}_{1i}$ and $\mathbf{p}_{2i}$ before the collision event and $\mathbf{p}_{1f}$ and $\mathbf{p}_{2f}$ afterwards. But the sum of the momenta before and after the collision is conserved, as written in Eq. 7.3.

While the *total momentum* is conserved for a system of isolated colliding particles, the *mechanical energy* may or may not be conserved. If the mechanical energy (usually meaning the total kinetic energy) is the same before and after a collision, we say that the collision is **elastic**. Otherwise we say the collision is **inelastic**.

If two objects collide, stick together, and move off as a combined mass, we call this a **perfectly inelastic** collision. One can show that in such a collision more kinetic energy is lost than if the objects were to bounce off one another and move off separately.

When two particles undergo an *elastic* collision then we also know that

$$\tfrac{1}{2}m_1 v_{1i}^2 + \tfrac{1}{2}m_2 v_{2i}^2 = \tfrac{1}{2}m_1 v_{1f}^2 + \tfrac{1}{2}m_2 v_{2f}^2 \ .$$

In the special case of a one-dimensional elastic collision between masses m_1 and m_2 we can relate the final velocities to the initial velocities. The result is

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i} \tag{7.5}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i} \tag{7.6}$$

This result can be useful in solving a problem where such a collision occurs, but it is *not* a fundamental equation. So don't memorize it.

7.1.5 The Center of Mass

For a system of particles (that is, lots of 'em) there is a special point in space known as the **center of mass** which is of great importance in describing the overall motion of the system. This point is a weighted average of the positions of all the mass points.

If the particles in the system have masses m_1, m_2, $\dots m_N$, with total mass

$$\sum_i^N m_i = m_1 + m_2 + \cdots + m_N \equiv M$$

and respective positions $\mathbf{r_1}$, $\mathbf{r_2}$, $\dots$,$\mathbf{r_N}$, then the center of mass $\mathbf{r}_{\rm CM}$ is:

$$\mathbf{r}_{\rm CM} = \frac{1}{M}\sum_i^N m_i \mathbf{r}_i \tag{7.7}$$

which means that the x, y and z coordinates of the center of mass are

$$x_{\rm CM} = \frac{1}{M}\sum_i^N m_i x_i \qquad y_{\rm CM} = \frac{1}{M}\sum_i^N m_i y_i \qquad z_{\rm CM} = \frac{1}{M}\sum_i^N m_i z_i \tag{7.8}$$

For an extended object (i.e. a continuous distribution of mass) the definition of $\mathbf{r}_{\rm CM}$ is given by an *integral* over the mass elements of the object:

$$\mathbf{r}_{\rm CM} = \frac{1}{M}\int \mathbf{r}\, dm \tag{7.9}$$

which means that the x, y and z coordinates of the center of mass are now:

$$x_{\rm CM} = \frac{1}{M}\int x\, dm \qquad y_{\rm CM} = \frac{1}{M}\int y\, dm \qquad z_{\rm CM} = \frac{1}{M}\int z\, dm \tag{7.10}$$

When the particles of a system are in motion then in general their center of mass is also in motion. The velocity of the center of mass is a similar weighted average of the individual velocities:

$$\mathbf{v}_{\rm CM} = \frac{d\mathbf{r}_{\rm CM}}{dt} = \frac{1}{M}\sum_i^N m_i \mathbf{v}_i \tag{7.11}$$

In general the center of mass will accelerate; its acceleration is given by

$$\mathbf{a}_{\rm CM} = \frac{d\mathbf{v}_{\rm CM}}{dt} = \frac{1}{M}\sum_i^N m_i \mathbf{a}_i \tag{7.12}$$

If $\mathbf{P}$ is the total momentum of the system and M is the total mass of the system, then the motion of the center of mass is related to $\mathbf{P}$ by:

$$\mathbf{v}_{\rm CM} = \frac{\mathbf{P}}{M} \qquad \text{and} \qquad \mathbf{a}_{\rm CM} = \frac{1}{M}\frac{d\mathbf{P}}{dt}$$

7.1.6 The Motion of a System of Particles

A system of *many* particles (or an extended object) in general has a motion for which the description is very complicated, but it is possible to make a simple statement about the motion of its center of mass. Each of the particles in the system may feel forces from the other particles in the system, but it may also experience a net force from the (external) environment; we will denote this force by $\mathbf{F}_{\rm ext}$. We find that when we add up all the *external* forces acting on all the particles in a system, it gives the acceleration of the *center of mass* according to:

$$\sum_i^N \mathbf{F}_{{\rm ext},\,i} = M\mathbf{a}_{\rm CM} = \frac{d\mathbf{P}}{dt} \tag{7.13}$$

Here, M is the total mass of the system; $\mathbf{F}_{{\rm ext},\,i}$ is the external force acting on particle i.

In words, we can express this result in the following way: For a system of particles, the center of mass moves as if it were a *single* particle of mass M moving under the influence of the sum of the external forces.

7.2 Worked Examples

7.2.1 Linear Momentum

1. A $3.00\,{\rm kg}$ particle has a velocity of $(3.0\mathbf{i} - 4.0\mathbf{j})\,\frac{\rm m}{\rm s}$. Find its x and y components of momentum and the magnitude of its total momentum.

Using the definition of momentum and the given values of m and $\mathbf{v}$ we have:

$$\mathbf{p} = m\mathbf{v} = (3.00\,{\rm kg})(3.0\mathbf{i} - 4.0\mathbf{j})\,\tfrac{\rm m}{\rm s} = (9.0\mathbf{i} - 12.\mathbf{j})\,\tfrac{\rm kg\cdot m}{\rm s}$$

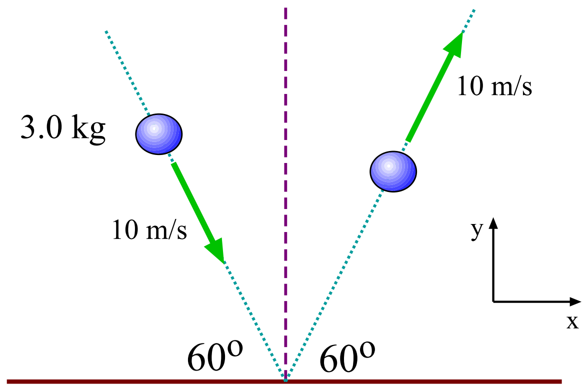

Figure 7.1: Ball bounces off wall in Example 3.

So the particle has momentum components

$$p_x = +9.0\,\frac{\text{kg}\cdot\text{m}}{\text{s}} \qquad \text{and} \qquad p_y = -12.\,\frac{\text{kg}\cdot\text{m}}{\text{s}}\ .$$

The magnitude of its momentum is

$$p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.0)^2 + (-12.)^2}\,\frac{\text{kg}\cdot\text{m}}{\text{s}} = 15.\,\frac{\text{kg}\cdot\text{m}}{\text{s}}$$

7.2.2 Impulse, Average Force

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2. A child bounces a superball on the sidewalk. The linear impulse delivered by the sidewalk is $2.00\,\text{N}\cdot\text{s}$ during the $\frac{1}{800}\,\text{s}$ of contact. What is the magnitude of the average force exerted on the ball by the sidewalk.

The magnitude of the change in momentum of (impulse delivered to) the ball is $|\Delta\mathbf{p}| = |\mathbf{I}| = 2.00\,\text{N}\cdot\text{s}$. (The *direction* of the impulse is upward, since the initial momentum of the ball was downward and the final momentum is upward.)

Since the time over which the force was acting was

$$\Delta t = \frac{1}{800}\,\text{s} = 1.25\times10^{-3}\,\text{s}$$

then from the definition of average force we get:

$$|\overline{\mathbf{F}}| = \frac{|\mathbf{I}|}{\Delta t} = \frac{2.00\,\text{N}\cdot\text{s}}{1.25\times10^{-3}\,\text{s}} = 1.60\times10^{3}\,\text{N}$$

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3. A $3.0\,\text{kg}$ steel ball strikes a wall with a speed of $10\,\frac{\text{m}}{\text{s}}$ at an angle of 60° with the surface. It bounces off with the same speed and angle, as shown in Fig. 7.1. If the ball is in contact with the wall for $0.20\,\text{s}$, what is the average force exerted on the wall by the ball?

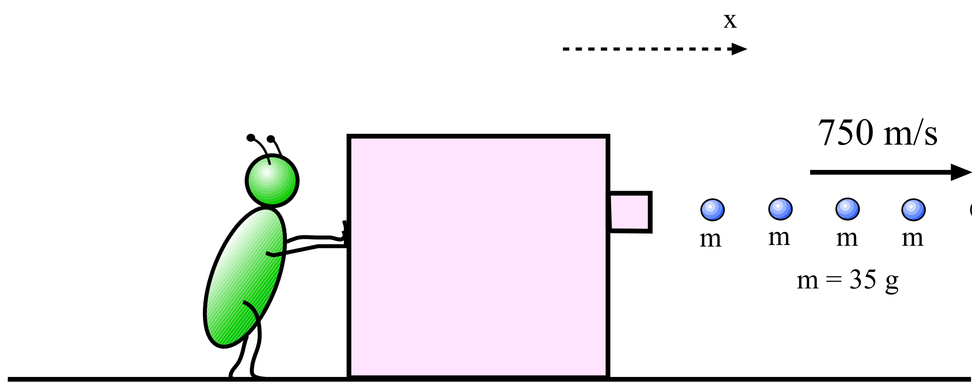

Figure 7.2: *Simplified* picture of a machine gun spewing out bullets. An external force is necessary to hold the gun in place!

The average force is defined as $\overline{\mathbf{F}} = \Delta\mathbf{p}/\Delta t$, so first find the change in momentum of the ball. Since the ball has the same speed before and after bouncing from the wall, it is clear that its x velocity (see the coordinate system in Fig. 7.1) stays the same and so the x momentum stays the same. But the y momentum *does* change. The initial y velocity is

$$v_{iy} = -(10\,\tfrac{\mathrm{m}}{\mathrm{s}})\sin 60^\circ = -8.7\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

and the final y velocity is

$$v_{fy} = +(10\,\tfrac{\mathrm{m}}{\mathrm{s}})\sin 60^\circ = +8.7\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

so the change in y momentum is

$$\Delta p_y = mv_{fy} - mv_{iy} = m(v_{fy} - v_{iy}) = (3.0\,\mathrm{kg})(8.7\,\tfrac{\mathrm{m}}{\mathrm{s}} - (-8.7\,\tfrac{\mathrm{m}}{\mathrm{s}})) = 52\,\tfrac{\mathrm{kg\cdot m}}{\mathrm{s}}$$

The average y force on the ball is

$$\overline{F}_y = \frac{\Delta p_y}{\Delta t} = \frac{I_y}{\Delta t} = \frac{(52\,\frac{\mathrm{kg\cdot m}}{\mathrm{s}})}{(0.20\,\mathrm{s})} = 2.6 \times 10^2\,\mathrm{N}$$

Since $\overline{\mathbf{F}}$ has no x component, the average force has magnitude $2.6 \times 10^2\,\mathrm{N}$ and points in the y direction (away from the wall).

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4. A machine gun fires $35.0\,\mathrm{g}$ bullets at a speed of $750.0\,\frac{\mathrm{m}}{\mathrm{s}}$. If the gun can fire $200\,\mathrm{bullets/\,min}$, what is the average force the shooter must exert to keep the gun from moving?

Whoa! Lots of things happening here. Let's draw a diagram and try to sort things out. Such a picture is given in Fig. 7.2.

The gun interacts with the bullets; it exerts a brief, strong force on each of the bullets which in turn exerts an "equal and opposite" force on the gun. The gun's force changes the bullet's momentum from *zero* (as they are initially at rest) to the final value of

$$p_f = mv = (0.0350\,\mathrm{kg})(750\,\tfrac{\mathrm{m}}{\mathrm{s}}) = 26.2\,\tfrac{\mathrm{kg\cdot m}}{\mathrm{s}}\ .$$

so this is also the *change* in momentum for each bullet.

Now, since 200 bullets are fired every minute (60 s), we should count the interaction time as the time to fire *one* bullet,

$$\Delta t = \frac{60\,\mathrm{s}}{200} = 0.30\,\mathrm{s}$$

because every 0.30 s, a firing occurs again, and the *average* force that we compute will be valid for a length of time for which many bullets are fired. So the average force of the gun on the bullets is

$$\overline{F}_x = \frac{\Delta p_x}{\Delta t} = \frac{26.2\,\frac{\mathrm{kg\cdot m}}{\mathrm{s}}}{0.30\,\mathrm{s}} = 87.5\,\mathrm{N}$$

From Newton's Third Law, there must an average backwards force *of the bullets on the gun* of magnitude 87.5 N. If there were no other forces acting on the gun, it would accelerate backward! To keep the gun in place, the shooter (or the gun's mechanical support) must exert a force of 87.5 N in the forward direction.

We can also work with the numbers as follows: In one minute, 200 bullets were fired, and a *total* momentum of

$$P = (200)(26.2\,\tfrac{\mathrm{kg\cdot m}}{\mathrm{s}}) = 5.24 \times 10^3\,\tfrac{\mathrm{kg\cdot m}}{\mathrm{s}}$$

was imparted to them. So during this time period (60 seconds!) the average force on the whole set of bullets was

$$\overline{F}_x = \frac{\Delta P}{\Delta t} = \frac{5.24 \times 10^3\,\frac{\mathrm{kg\cdot m}}{\mathrm{s}}}{60.0\,\mathrm{s}} = 87.5\,\mathrm{N}\ .$$

As before, this is also the average backwards force of the *bullets on the gun* and the force required to keep the gun in place.

7.2.3 Collisions

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5. A 10.0 g bullet is stopped in a block of wood ($m = 5.00\,\mathrm{kg}$). The speed of the bullet–plus–wood combination immediately after the collision is $0.600\,\frac{\mathrm{m}}{\mathrm{s}}$. What was the original speed of the bullet?

A picture of the collision just before and after the bullet (quickly) embeds itself in the wood is given in Fig. 7.3. The bullet has some initial speed v_0 (we don't know what it is.)

The collision (and embedding of the bullet) takes place very rapidly; for that brief time the bullet and block essentially form an isolated system because any external forces (say, from friction from the surface) will be of no importance compared to the *enormous* forces between the bullet and the block. So the total momentum of the system will be conserved; it is the same before and after the collision.

In this problem there is only motion along the x axis, so we only need the condition that the total x momentum (P_x) is conserved.

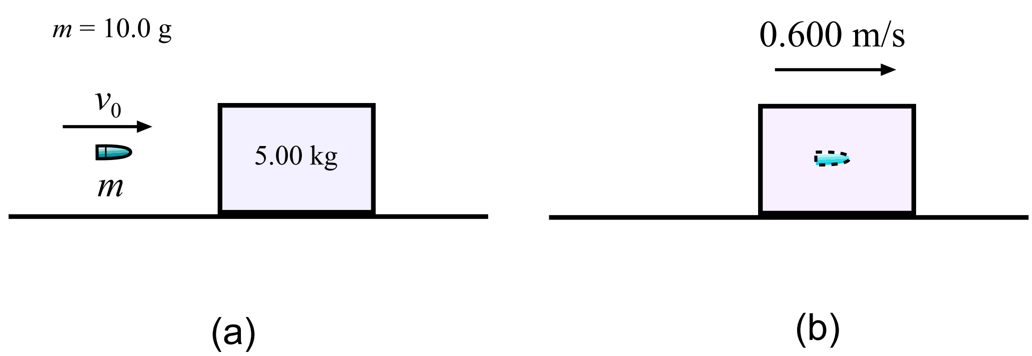

Figure 7.3: Collision in Example 5. (a) Just before the collision. (b) Just after.

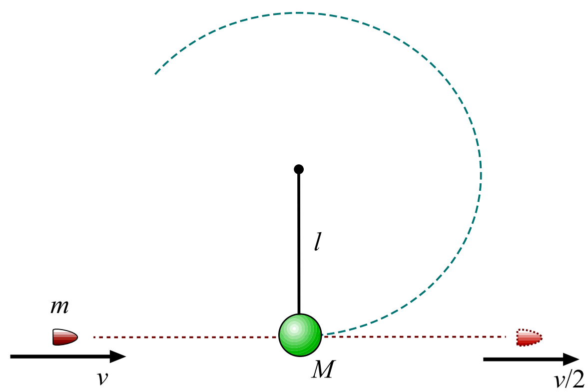

Figure 7.4: Bullet passes through a pendulum bob and emerges with half its original speed; the bob barely swings through a complete circle!

Just before the collision, only the bullet (with mass m) is in motion and its x velocity is v_0. So the initial momentum is

$$P_{i,x} = mv_0 = (10.0 \times 10^{-3}\,\mathrm{kg})v_0$$

Just after the collision, the bullet–block combination, with its mass of $M + m$ has an x velocity of $0.600\,\frac{\mathrm{m}}{\mathrm{s}}$. So the final momentum is

$$P_{f,x} = (M+m)v = (5.00\,\mathrm{kg} + 10.0 \times 10^{-3}\,\mathrm{kg})(0.600\,\tfrac{\mathrm{m}}{\mathrm{s}}) = 3.01\,\tfrac{\mathrm{kg\cdot m}}{\mathrm{s}}$$

Since $P_{i,x} = P_{f,x}$, we get:

$$(10.0 \times 10^{-3}\,\mathrm{kg})v_0 = 3.01\,\tfrac{\mathrm{kg\cdot m}}{\mathrm{s}} \qquad \Longrightarrow \qquad v_0 = 301\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

The initial speed of the bullet was $301\,\frac{\mathrm{m}}{\mathrm{s}}$.

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6. As shown in Fig. 7.4, a bullet of mass m and speed v passes completely through a pendulum bob of mass M. The bullet emerges with a speed $v/2$. The pendulum bob is suspended by a stiff rod of length ℓ and negligible mass. What

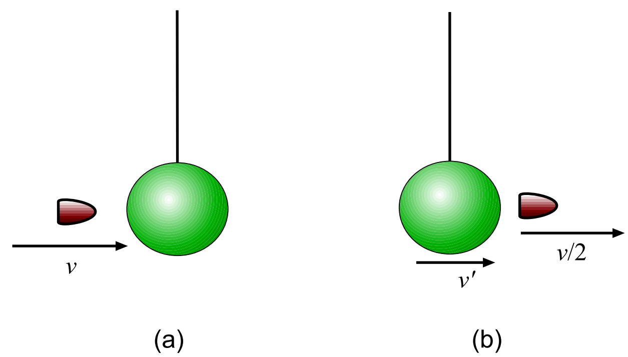

Figure 7.5: Collision of the bullet with the pendulum bob. (a) Just before the collision. (b) Just after. The bullet has gone through the bob, which has acquired a velocity v'.

is the minimum value of v such that the pendulum bob will barely swing through a complete vertical circle?

Whoa! There's a hell of a lot of things going on in this problem. Let's try to sort them out.

We break things down into a sequence of events: First, the bullet has a very rapid, very strong interaction with the pendulum bob, where is quickly passes through, imparting a velocity to the bob which at first will have a horizontal motion. Secondly, the bob swings upward and, as we are told, will get up to the top of the vertical circle.

We show the collision in Fig. 7.5. In this rapid interaction there are no net external forces acting on the system that we need to worry about. So its total momentum will be conserved. The total horizontal momentum before the collision is

$$P_{i,x} = mv + 0 = mv$$

If after the collision the bob has velocity v', then the total momentum is

$$P_{f,x} = m\left(\frac{v}{2}\right) + Mv'$$

Conservation of momentum, $P_{i,x} = P_{f,x}$ gives

$$mv = m\left(\frac{v}{2}\right) + Mv' \qquad \Longrightarrow \qquad Mv' = m\left(\frac{v}{2}\right)$$

and so:

$$v' = \frac{1}{M}\frac{mv}{2} = \frac{mv}{2M} \tag{7.14}$$

Now consider the trip of the pendulum bob up to the top of the circle (it *must* get to the top, by assumption). There are no friction–type forces acting on the system as M moves, so *mechanical energy is conserved*.

If we measure height from the bottom of the swing, then the initial potential energy is zero while the initial kinetic energy is

$$K_i = \tfrac{1}{2}M\left(v'\right)^2 \ .$$

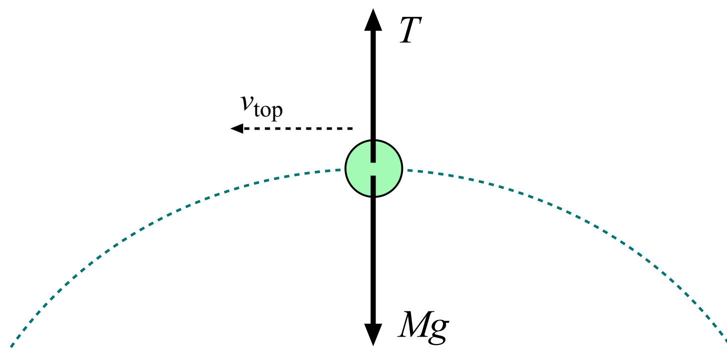

Figure 7.6: Forces acting on pendulum bob M at the top of the swing.

Now suppose at the top of the swing mass M has speed $v_{\rm top}$. Its height is 2ℓ and its potential energy is $Mg(2\ell)$ so that its final energy is

$$E_f = \tfrac{1}{2}Mv_{\rm top}^2 + 2Mg\ell$$

so that conservation of energy gives:

$$\tfrac{1}{2}M\left(v'\right)^2 = \tfrac{1}{2}Mv_{\rm top}^2 + 2Mg\ell \qquad (7.15)$$

What do we know about $v_{\rm top}$? A drawing of the forces acting on M at the top of the swing is shown in Fig. 7.6. Gravity pulls down with a force Mg. There may be a force from the suspending rod; here, I've happened to draw it pointing upward. *Can* this force point upward? Yes it can... we need to read the problem carefully. It said the bob was suspended by a *stiff rod* and such an object can exert a force (still called the tension T) *inward or outward* along its length. (A string can only pull inward.) The bob is moving on a circular path with (instantaneous) speed $v_{\rm top}$ so the net force on it points *downward* and has magnitude $Mv_{\rm top}^2/\ell$:

$$Mg - T = \frac{Mv_{\rm top}^2}{\ell} \ .$$

Since T can be positive or negative, $v_{\rm top}$ can take on any value. It could be zero. What condition are we looking for which corresponds to the smallest value of the bullet speed v?

We note that as v gets bigger, so does v' (the bob's initial speed). As v' increases, so does $v_{\rm top}$, as we see from conservation of energy. But it is entirely possible for $v_{\rm top}$ to be zero, and *that* will give the smallest possible value of v. That would correspond to the case where M picked up enough speed to *just barely* make it to the top of the swing. (And when the bob goes past the top point then gravity moves it along through the full swing.)

So with $v_{\rm top} = 0$ then Eq. 7.15 gives us

$$\tfrac{1}{2}M\left(v'\right)^2 = 2Mg\ell \qquad \Longrightarrow \qquad v'^2 = 4g\ell$$

and:

$$v' = \sqrt{4g\ell}$$

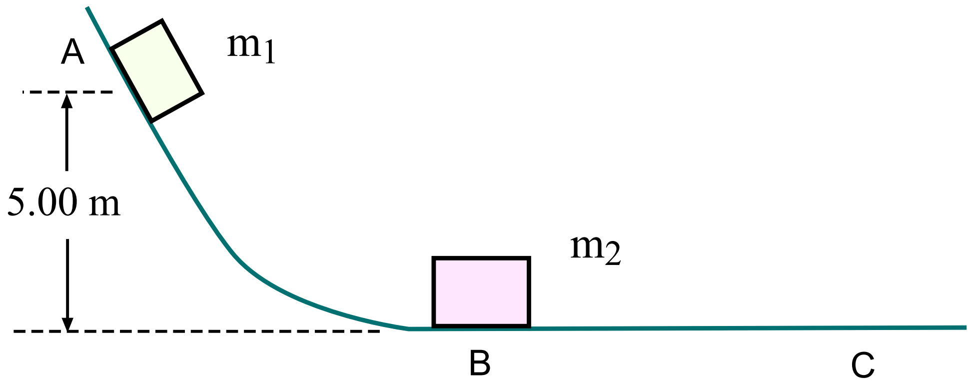

Figure 7.7: Frictionless track, for Example 7. Mass m_1 is released and collides elastically with mass m_2.

and putting this result back into Eq. 7.15, we have

$$\sqrt{4g\ell} = \frac{1}{M}\frac{mv}{2} = \frac{mv}{2M} \ .$$

Finally, solve for v:

$$v = \frac{2M}{m}\sqrt{4g\ell} = \frac{4M\sqrt{g\ell}}{m}$$

The minimum value of v required to do the job is $v = 4M\sqrt{g\ell}/m$.

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7. Consider a frictionless track ABC as shown in Fig. 7.7. A block of mass $m_1 = 5.001\,\mathrm{kg}$ is released from A. It makes a head–on elastic collision with a block of mass $m_2 = 10.0\,\mathrm{kg}$ at B, initially at rest. Calculate the maximum height to which m_1 rises after the collision.

Whoa! What is this problem talking about??

We release mass m_1; it slides down to the slope, picking up speed, until it reaches B. At B it makes a collision with mass m_2, and we are told it is an *elastic* collison. The last sentence in the problem implies that in this collision m_1 will reverse its direction of motion and head *back up* the slope to some maximum height. We would also guess that m_2 will be given a forward velocity.

This sequence is shown in Fig. 7.8. First we think about the instant of time just before the collision. Mass m_1 has velocity v_{1i} and mass m_2 is still stationary. How can we find v_{1i}? We can use the fact that *energy is conserved* as m_1 slides down the smooth (frictionless) slope. At the top of the slope m_1 had some potential energy, $U = m_1gh$ (with $h = 5.00\,\mathrm{m}$) which is changed to kinetic energy, $K = \frac{1}{2}m_1v_{1i}^2$ when it reaches the bottom. Conservation of energy gives us:

$$m_1gh = \tfrac{1}{2}mv_{1i}^2 \qquad \Longrightarrow \qquad v_{1i}^2 = 2gh = 2(9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(5.00\,\mathrm{m}) = 98.0\,\tfrac{\mathrm{m}^2}{\mathrm{s}^2}$$

so that

$$v_{1i} = +9.90\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

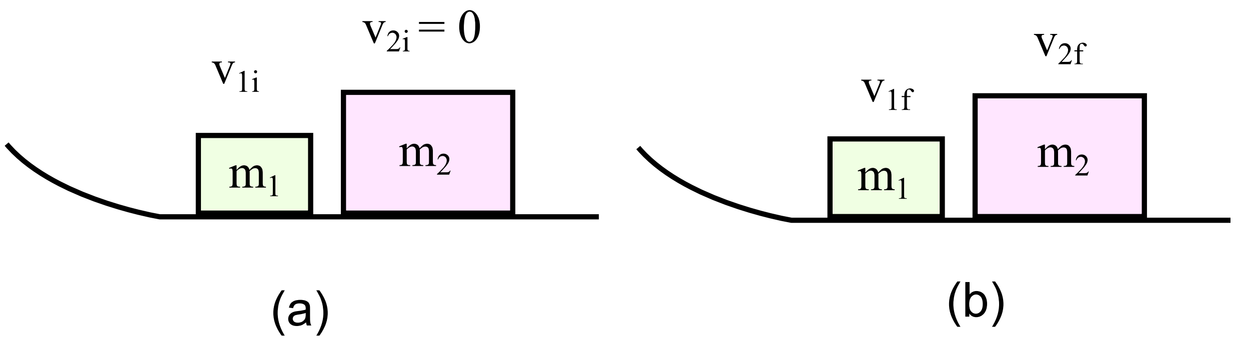

Figure 7.8: (a) Just before the collision; m_1 has acquired a velocity of v_{1i} from sliding down the slope. (b) Just after the collision; mass m_1 has *velocity* v_{1f} and mass m_2 has *velocity* v_{2f}.

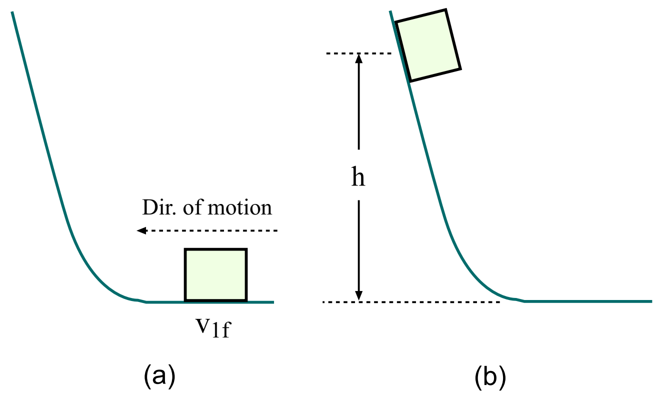

Figure 7.9: (a) After the collision, m_1 goes to the left and will move back up the slope. (b) After moving back up the slope, m_1 reaches some maximum height h.

We chose the *positive* value here since m_1 is obviously moving *forward* at the bottom of the slope. So m_1's velocity just before striking m_2 is $+9.90\,\frac{\mathrm{m}}{\mathrm{s}}$.

Now m_1 makes an elastic (one–dimensional) collision with m_2. What are the final velocities of the masses? For this we can use the result given in Eqs. 7.5 and 7.6, using $v_{2i} = 0$. We get:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} = \left(\frac{5.001\,\mathrm{kg} - 10.0\,\mathrm{kg}}{5.001\,\mathrm{kg} + 10.0\,\mathrm{kg}}\right)(+9.90\,\tfrac{\mathrm{m}}{\mathrm{s}}) = -3.30\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} = \left(\frac{2(5.001\,\mathrm{kg})}{5.001\,\mathrm{kg} + 10.0\,\mathrm{kg}}\right)(+9.90\,\tfrac{\mathrm{m}}{\mathrm{s}}) = +6.60\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

So after the collision, m_1 has a *velocity* of $-3.30\,\frac{\mathrm{m}}{\mathrm{s}}$; that is, it has *speed* $3.30\,\frac{\mathrm{m}}{\mathrm{s}}$ and it is now moving *to the left*. After the collision, m_2 has velocity $+6.60\,\frac{\mathrm{m}}{\mathrm{s}}$, so that it is moving to the right with speed $6.60\,\frac{\mathrm{m}}{\mathrm{s}}$.

Since m_1 is now moving to the left, it will head back up the slope. (See Fig. 7.9.) How high will it go? Once again, we can use energy conservation to give us the answer. For the trip back up the slope, the initial energy (all kinetic) is

$$E_i = K_i = \tfrac{1}{2}m(3.30\,\tfrac{\mathrm{m}}{\mathrm{s}})^2$$

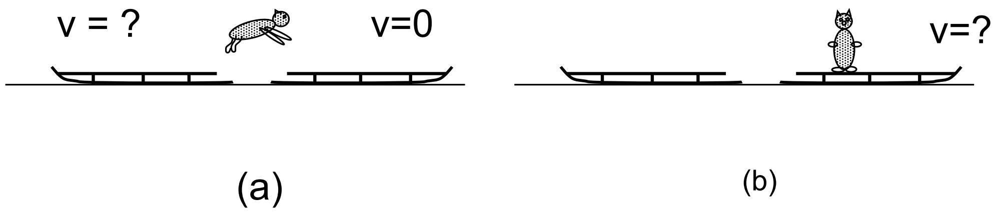

Figure 7.10: (a) Cat leaps from left sled to the right sled. What is new velocity of left sled? (b) Cat has landed on the right sled. What is its velocity now?

and when it reaches maximum height (h) its speed is zero, so its energy is the potential energy,

$$E_f = U_f = mgh$$

Conservation of energy, $E_i = E_f$ gives us:

$$\tfrac{1}{2}m(3.30\,\tfrac{\mathrm{m}}{\mathrm{s}})^2 = mgh \qquad \Longrightarrow \qquad h = \frac{(3.30\,\frac{\mathrm{m}}{\mathrm{s}})^2}{2g} = 0.556\,\mathrm{m}$$

Mass m_1 will travel back up the slope to a height of 0.556 m.

---

8. Two 22.7 kg ice sleds are placed a short distance apart, one directly behind the other, as shown in Fig. 7.10 (a). A 3.63 kg cat, standing on one sled, jumps across to the other and immediately back to the first. Both jumps are made at a speed on $3.05\,\frac{\mathrm{m}}{\mathrm{s}}$ relative to the ice. Find the final speeds of the two sleds.

We will let the x axis point to the right. In the initial picture (not shown) the cat is sitting on the left sled and both are motionless. Taking our system of interacting "particles" to be the cat and the left sled, the initial momentum of the system is $P = 0$.

After the cat has made its first jump, the velocity of this sled will be $v_{L,x}$, and the (final) total momentum of the system will be

$$P_f = (22.7\,\mathrm{kg})v_{L,x} + (3.63\,\mathrm{kg})(+3.05\,\tfrac{\mathrm{m}}{\mathrm{s}})$$

Note, we are using velocities with respect to the ice, and that is how we were given the velocity of the cat. Now as there are no net external forces, the momentum of this system is conserved. This gives us:

$$0 = (22.7\,\mathrm{kg})v_{L,x} + (3.63\,\mathrm{kg})(+3.05\,\tfrac{\mathrm{m}}{\mathrm{s}})$$

with which we easily solve for $v_{L,x}$:

$$v_{L,x} = -\frac{(3.63\,\mathrm{kg})(+3.05\,\frac{\mathrm{m}}{\mathrm{s}})}{(22.7\,\mathrm{kg})} = -0.488\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

so that the left sled moves at a speed of $0.488\,\frac{\mathrm{m}}{\mathrm{s}}$ *to the left* after the cat's first jump.

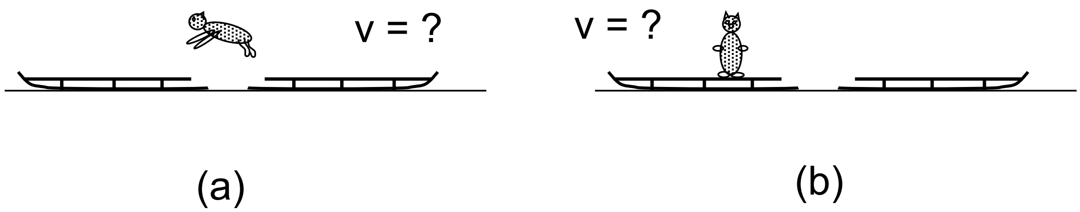

Figure 7.11: (a) Cat leaps from right sled back to left sled. What is new velocity of right sled? (b) Cat has landed back on left sled. What is *its* velocity now? Note that this is basically the mirror image of the previous figure, so I only had to draw it once! Hah!

The cat lands on the right sled and after landing it moves with the same velocity as that sled; the collision here is completely inelastic. For this part of the problem, the system of "interacting particles" we consider is *the cat and the right sled*. (The left sled does not interact with *this* system.) The initial momentum of this system is just that of the cat,

$$P_i = (3.63\,\mathrm{kg})(+3.05\,\tfrac{\mathrm{m}}{\mathrm{s}}) = 11.1\,\tfrac{\mathrm{kg\cdot m}}{\mathrm{s}}$$

If the final velocity of both cat and sled is $v_{R,x}$ then the final momentum is

$$P_f = (22.7\,\mathrm{kg} + 3.63\,\mathrm{kg})v_{R,x} = (26.3\,\mathrm{kg})v_{R,x}$$

(The cat and sled move as one mass, so we can just add their individual masses.) Conservation of momentum of this system, $P_i = P_f$ gives

$$11.1\,\tfrac{\mathrm{kg\cdot m}}{\mathrm{s}} = (26.3\,\mathrm{kg})v_{R,x}$$

so

$$v_{R,x} = \frac{\left(11.1\,\frac{\mathrm{kg\cdot m}}{\mathrm{s}}\right)}{(26.3\,\mathrm{kg})} = 0.422\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

Now we have the velocities of both sleds as they are pictured in Fig. 7.10 (b).

And now the cat makes a jump back to the left sled, as shown in Fig. 7.11 (a). Again, we take the system to be the cat and the right sled. Its initial momentum is

$$P_i = (22.7\,\mathrm{kg} + 3.63\,\mathrm{kg})(0.422\,\tfrac{\mathrm{m}}{\mathrm{s}}) = 11.1\,\tfrac{\mathrm{kg\cdot m}}{\mathrm{s}}$$

Now after the cat leaps, the velocity of the cat (with respect to the ice) is $-3.05\,\frac{\mathrm{m}}{\mathrm{s}}$, as specified in the problem. If the velocity of the right sled after the leap is $v'_{R,x}$ then the final momentum of the system is

$$P_f = (3.63\,\mathrm{kg})(-3.05\,\tfrac{\mathrm{m}}{\mathrm{s}}) + (22.7\,\mathrm{kg})v'_{R,x}$$

Conservation of momentum for the system, $P_i = P_f$, gives

$$11.1\,\tfrac{\mathrm{kg\cdot m}}{\mathrm{s}} = (3.63\,\mathrm{kg})(-3.05\,\tfrac{\mathrm{m}}{\mathrm{s}}) + (22.7\,\mathrm{kg})v'_{R,x}$$

so that we can solve for $v'_{R,x}$:

$$v'_{R,x} = \frac{(11.1\,\frac{\text{kg}\cdot\text{m}}{\text{s}} + 11.1\,\frac{\text{kg}\cdot\text{m}}{\text{s}})}{(22.7\,\text{kg})} = 0.975\,\frac{\text{m}}{\text{s}}$$

so during its second leap the cat makes the right sled go faster!

Finally, for the cat's landing on the left sled we consider the (isolated) system of the cat and the left sled. We already have the velocities of the cat and sled at this time; its initial momentum is

$$P_i = (22.7\,\text{kg})(-0.488\,\tfrac{\text{m}}{\text{s}}) + (3.63\,\text{kg})(-3.05\,\tfrac{\text{m}}{\text{s}}) = -22.1\,\tfrac{\text{kg}\cdot\text{m}}{\text{s}}\ .$$

After the cat has landed on the sled, it is moving with the same velocity as the sled, which we will call $v'_{L,x}$. Then the final momentum of the system is

$$P_f = (22.7\,\text{kg} + 3.63\,\text{kg})v'_{L,x} = (26.3\,\text{kg})v'_{L,x}$$

And momentum conservation for *this* collision gives

$$-22.1\,\tfrac{\text{kg}\cdot\text{m}}{\text{s}} = (26.3\,\text{kg})v'_{L,x}$$

and then

$$v'_{L,x} = \frac{(-22.1\,\frac{\text{kg}\cdot\text{m}}{\text{s}})}{(26.3\,\text{kg})} = -0.842\,\frac{\text{m}}{\text{s}}$$

Summing up, the final velocities of the sleds (after the cat is done jumping) are:

$$\text{Left Sled:} \qquad v'_{L,x} = -0.842\,\tfrac{\text{m}}{\text{s}}$$

$$\text{Right Sled:} \qquad v'_{r,x} = +0.975\,\tfrac{\text{m}}{\text{s}}$$

7.2.4 Two–Dimensional Collisions

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9. An unstable nucleus of mass $17 \times 10^{-27}\,\text{kg}$ initially at rest disintegrates into three particles. One of the particles, of mass $5.0 \times 10^{-27}\,\text{kg}$, moves along the y axis with a speed of $6.0 \times 10^{6}\,\frac{\text{m}}{\text{s}}$. Another particle of mass $8.4 \times 10^{-27}\,\text{kg}$, moves along the x axis with a speed of $4.0 \times 10^{6}\,\frac{\text{m}}{\text{s}}$. Find (a) the velocity of the third particle and (b) the total energy given off in the process.

(a) First, draw a picture of what is happening! Such a picture is given in Fig. 7.12. In the most general sense of the word, this is indeed a "collision", since it involves the rapid interaction of a few isolated particles.

The are no external forces acting on the particles involved in the disintegration; the total momentum of the system is conserved. The parent nucleus is *at rest*, so that the total momentum was (and remains) zero: $\mathbf{P}_i = \mathbf{0}$.

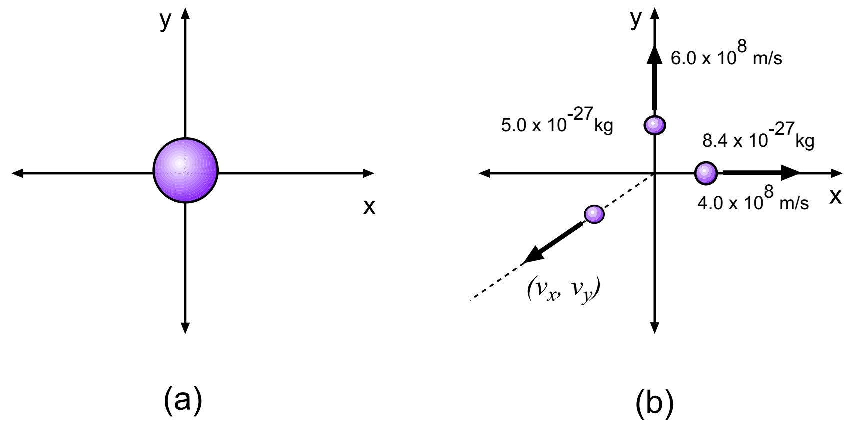

Figure 7.12: Nucleus disintegrates in Example 9. (a) Before the split; nucleus is at rest. (b) Afterwards; three pieces fly off in different directions.

Afterwards, the system consists of three particles; for two of these particles, we are given the masses and velocities. We are not given the mass of the third piece, but since we were given the mass of the parent nucleus, we might think that we can use the fact that the masses must sum up to the same value before and after the reaction to find it. In fact relativity tells us that masses don't really add in this way and when nuclei break up there *is* a measurable mass difference, but it is small enough that we can safely ignore it in this problem. So we would say that *mass* is conserved, and if m is the mass of the unknown fragment, we get:

$$17 \times 10^{-27}\,\mathrm{kg} = 5.0 \times 10^{-27}\,\mathrm{kg} + 8.4 \times 10^{-27}\,\mathrm{kg} + m$$

so that

$$m = 3.6 \times 10^{-27}\,\mathrm{kg}\ .$$

We will let the velocity components of the third fragment be v_x and v_y. Then the total x momentum after the collision is

$$P_{f,x} = (8.4 \times 10^{-27}\,\mathrm{kg})(4.0 \times 10^{6}\,\tfrac{\mathrm{m}}{\mathrm{s}}) + (3.6 \times 10^{-27}\,\mathrm{kg})v_x$$

Using $P_{i,x} = 0 = P_{f,x}$, we find:

$$(8.4 \times 10^{-27}\,\mathrm{kg})(4.0 \times 10^{6}\,\tfrac{\mathrm{m}}{\mathrm{s}}) + (3.6 \times 10^{-27}\,\mathrm{kg})v_x = 0$$

which easily gives:

$$v_x = -9.33 \times 10^{6}\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

Similarly, the total y momentum after the collision is:

$$P_{f,y} = (5.0 \times 10^{-27}\,\mathrm{kg})(6.0 \times 10^{6}\,\tfrac{\mathrm{m}}{\mathrm{s}}) + (3.6 \times 10^{-27}\,\mathrm{kg})v_y$$

and using $P_{i,y} = 0 = P_{f,y}$, we have:

$$(5.0 \times 10^{-27}\,\mathrm{kg})(6.0 \times 10^{6}\,\tfrac{\mathrm{m}}{\mathrm{s}}) + (3.6 \times 10^{-27}\,\mathrm{kg})v_y = 0$$

which gives

$$v_y = -8.33 \times 10^6 \ \tfrac{\text{m}}{\text{s}}$$

This really does specify the velocity of the third fragment (as requested), but it is also useful to express it as a magnitude and direction. The *speed* of the third fragment is

$$v = \sqrt{(-9.33 \times 10^6 \ \tfrac{\text{m}}{\text{s}})^2 + (-8.33 \times 10^6 \ \tfrac{\text{m}}{\text{s}})^2} = 1.25 \times 10^7 \ \tfrac{\text{m}}{\text{s}}$$

and its direction θ (measured counterclockwise from the x axis) is given by

$$\tan\theta = \left(\frac{-8.33}{-9.33}\right) = 0.893$$

Realizing that θ must lie in the third quadrant, we find:

$$\theta = \tan^{-1}(0.893) - 180^\circ = -138^\circ \ .$$

(b) What is the gain in energy by the system for this disintegration? By this we mean the gain in kinetic energy. Initially, the system has *no* kinetic energy. After the breakup, the kinetic energy is the sum of $\frac{1}{2}mv^2$ for all the particles, namely

$$\begin{aligned} K_f &= \tfrac{1}{2}(5.0 \times 10^{-27}\,\text{kg})(6.0 \times 10^6 \ \tfrac{\text{m}}{\text{s}})^2 + \tfrac{1}{2}(8.4 \times 10^{-27}\,\text{kg})(4.0 \times 10^6 \ \tfrac{\text{m}}{\text{s}})^2 \\ &\qquad + \tfrac{1}{2}(3.6 \times 10^{-27}\,\text{kg})(1.25 \times 10^7 \ \tfrac{\text{m}}{\text{s}})^2 \\ &= 4.38 \times 10^{-13}\,\text{J} \end{aligned}$$

We might say that the process gives off 4.38×10^{-13} J of energy.

---

10. A billiard ball moving at $5.00\ \frac{\text{m}}{\text{s}}$ strikes a stationary ball of the same mass. After the collision, the first ball moves at $4.33\ \frac{\text{m}}{\text{s}}$ at an angle of 30.0° with respect to the original line of motion. Assuming an elastic collision (and ignoring friction and rotational motion), find the struck ball's velocity.

The collision is diagrammed in Fig. 7.13. We don't know the final speed of the struck ball; we will call it v, as in Fig. 7.13(b). We don't know the final direction of motion of the struck ball; we will let it be some angle θ, measured below the x axis, also as shown in Fig. 7.13(b).

Since we are dealing with a "collision" between the two objects, we know that the total momentum of the system is conserved. So the x and y components of the total momentum is the same before and after the collision.

Suppose we let the x and y components of the struck ball's final velocity be v_x and v_y, respectively. Then the condition that the total x momentum be conserved gives us:

$$m(5.00\ \tfrac{\text{m}}{\text{s}}) + 0 = m(4.33\ \tfrac{\text{m}}{\text{s}})\cos 30.0^\circ + mv_x$$

(The struck ball has *no* momentum initially; after the collision, the incident ball has an x velocity of $(4.33\ \frac{\text{m}}{\text{s}})\cos 30.0^\circ$.)

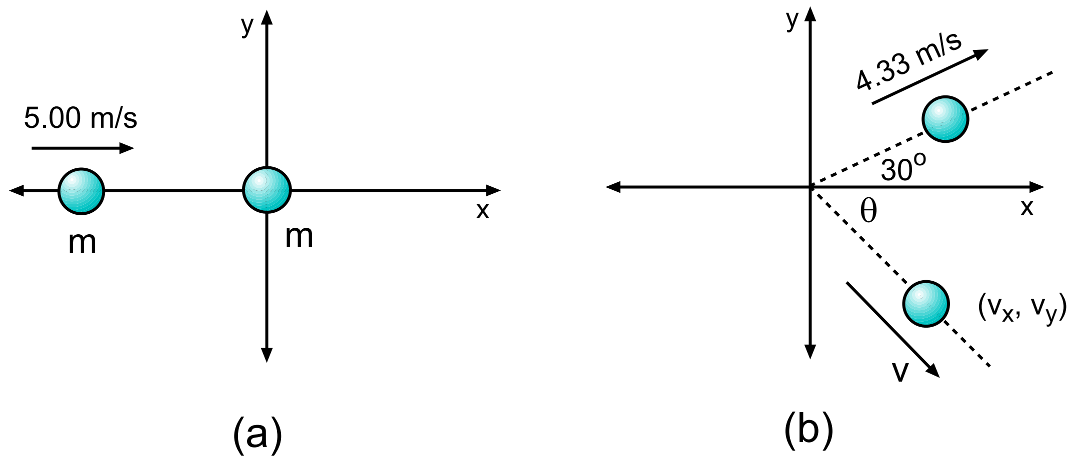

Figure 7.13: Collision in Example 10. (a) Before the collision. (b) After the collision.

Luckily, the m's cancel out of this equation and we can solve for v_x:

$$(5.00\,\tfrac{\mathrm{m}}{\mathrm{s}}) = (4.33\,\tfrac{\mathrm{m}}{\mathrm{s}})\cos 30.0^\circ + v_x$$

and then:

$$v_x = (5.00\,\tfrac{\mathrm{m}}{\mathrm{s}}) - (4.33\,\tfrac{\mathrm{m}}{\mathrm{s}})\cos 30.0^\circ = 1.25\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

We can similarly find v_y by using the condition that the total y momentum be conserved in the collision. This gives us:

$$0 + 0 = m(4.33\,\tfrac{\mathrm{m}}{\mathrm{s}})\sin 30.0^\circ + mv_y$$

which gives

$$v_y = -(4.33\,\tfrac{\mathrm{m}}{\mathrm{s}})\sin 30.0^\circ = -2.16\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

Now that we have the components of the final velocity we can find the speed and direction of motion. The speed is:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(1.25\,\tfrac{\mathrm{m}}{\mathrm{s}})^2 + (-2.16\,\tfrac{\mathrm{m}}{\mathrm{s}})^2} = 2.50\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

and the direction is found from

$$\tan\theta = \frac{v_y}{v_x} = \frac{-2.16\,\frac{\mathrm{m}}{\mathrm{s}}}{1.25\,\frac{\mathrm{m}}{\mathrm{s}}} = -1.73 \qquad \Longrightarrow \qquad \theta = \tan^{-1}(-1.73) = -60^\circ$$

So the struck ball moves off with a speed of $2.50\,\frac{\mathrm{m}}{\mathrm{s}}$ at an angle of 60° downward from the x axis.

This really completes the problem but we notice something strange here: We were given *more* information about the collision than we used. We were also told that the collision was *elastic*, meaning that the total kinetic energy of the system was the same before and after the collision. Since we now have all of the speeds, we can *check* this.

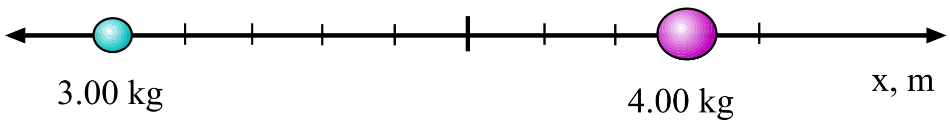

Figure 7.14: Masses and positions for Example 11.

The total kinetic energy before the collision was

$$K_i = \tfrac{1}{2}m(5.00\,\tfrac{\mathrm{m}}{\mathrm{s}})^2 = m(12.5\,\tfrac{\mathrm{m}^2}{\mathrm{s}^2})\ .$$

The total kinetic energy after the collision was

$$K_f = \tfrac{1}{2}m(4.33\,\tfrac{\mathrm{m}}{\mathrm{s}})^2 + \tfrac{1}{2}m(2.50\,\tfrac{\mathrm{m}}{\mathrm{s}})^2 = m(12.5\,\tfrac{\mathrm{m}^2}{\mathrm{s}^2})$$

so that K_i is the same as K_f; the statement that the collision is elastic is *consistent* with the other data; but in this case we were given *too much* information in the problem!

7.2.5 The Center of Mass

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11. A $3.00\,\mathrm{kg}$ **particle is located on the** x **axis at** $x = -5.00\,\mathrm{m}$ **and a** $4.00\,\mathrm{kg}$ **particle is on the** x **axis at** $x = 3.00\,\mathrm{m}$**. Find the center of mass of this two–particle system.**

The masses are shown in Fig. 7.14. There is only one coordinate (x) and two mass points to consider here; using the definition of x_{CM}, we find:

$$\begin{aligned} x_{\mathrm{CM}} &= \frac{m_1x_1 + m_2x_2}{m_1+m_2} \\ &= \frac{(3.00\,\mathrm{kg})(-5.00\,\mathrm{m}) + (4.00\,\mathrm{kg})(3.00\,\mathrm{m})}{(3.00\,\mathrm{kg} + 4.00\,\mathrm{kg})} \\ &= -0.429\,\mathrm{m} \end{aligned}$$

The center of mass is located at $x = -0.429\,\mathrm{m}$.

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12. An old Chrysler with mass $2400\,\mathrm{kg}$ **is moving along a straight stretch of road at** $80\,\mathrm{km/h}$**. It is followed by a Ford with mass** $1600\,\mathrm{kg}$ **moving at** $60\,\mathrm{km/h}$**. How fast is the center of mass of the two cars moving?**

The cars here have 1–dimensional motion along (say) the x axis.
From Eq. 7.11 we see that the velocity of their center of mass is given by:

$$v_{\mathrm{CM},\,x} = \frac{1}{M}\sum_i^N m_i v_{i,\,x} = \frac{m_1v_{1x} + m_2v_{2x}}{m_1+m_2}$$

Plugging in the masses and velocities of the two cars, we find

$$v_{\mathrm{CM},\,x} = \frac{(2400\,\mathrm{kg})\left(80\,\frac{\mathrm{km}}{\mathrm{h}}\right) + (1600\,\mathrm{kg})\left(60\,\frac{\mathrm{km}}{\mathrm{h}}\right)}{(2400\,\mathrm{kg} + 1600\,\mathrm{kg})} = 72\,\frac{\mathrm{km}}{\mathrm{h}}$$

In units of $\frac{\mathrm{m}}{\mathrm{s}}$ this is

$$72\,\frac{\mathrm{km}}{\mathrm{h}}\left(\frac{10^3\,\mathrm{m}}{1\,\mathrm{km}}\right)\left(\frac{1\,\mathrm{h}}{3600\,\mathrm{s}}\right) = 20\,\frac{\mathrm{m}}{\mathrm{s}}$$

7.2.6 The Motion of a System of Particles

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13. A $2.0\,\mathrm{kg}$ particle has a velocity of $\mathbf{v}_1 = (2.0\mathbf{i} - 3.0\mathbf{j})\,\frac{\mathrm{m}}{\mathrm{s}}$, and a $3.0\,\mathrm{kg}$ particle has a velocity $(1.0\mathbf{i} + 6.0\mathbf{j})\,\frac{\mathrm{m}}{\mathrm{s}}$. Find (a) the velocity of the center of mass and (b) the total momentum of the system.

(a) We are given the masses and velocity components for the two particles. Then writing out the x and y components of Eq. 7.11 we find:

$$\begin{aligned} v_{\mathrm{CM},x} &= \frac{(m_1 v_{1x} + m_2 v_{2x})}{(m_1 + m_2)} \\ &= \frac{(2.0\,\mathrm{kg})(2.0\,\frac{\mathrm{m}}{\mathrm{s}}) + (3.0\,\mathrm{kg})(1.0\,\frac{\mathrm{m}}{\mathrm{s}})}{(2.0\,\mathrm{kg} + 3.0\,\mathrm{kg})} \\ &= 1.4\,\frac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

$$\begin{aligned} v_{\mathrm{CM},y} &= \frac{(m_1 v_{1y} + m_2 v_{2y})}{(m_1 + m_2)} \\ &= \frac{(2.0\,\mathrm{kg})(-3.0\,\frac{\mathrm{m}}{\mathrm{s}}) + (3.0\,\mathrm{kg})(6.0\,\frac{\mathrm{m}}{\mathrm{s}})}{(2.0\,\mathrm{kg} + 3.0\,\mathrm{kg})} \\ &= 2.4\,\frac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

The velocity of the center of mass of the two–particle system is

$$\mathbf{v}_{\mathrm{CM}} = (1.4\mathbf{i} + 2.4\mathbf{j})\,\frac{\mathrm{m}}{\mathrm{s}}$$

(b) The total momentum of a system of particles is related to the velocity of the center of mass by $\mathbf{P} = M\mathbf{v}_{\mathrm{CM}}$ so we can use the answer from part (a) to get:

$$\begin{aligned} \mathbf{P} &= M\mathbf{v}_{\mathrm{CM}} = (2.0\,\mathrm{kg} + 3.0\,\mathrm{kg})((1.4\mathbf{i} + 2.4\mathbf{j})\,\frac{\mathrm{m}}{\mathrm{s}}) \\ &= (7.00\mathbf{i} + 12.0\mathbf{j})\,\frac{\mathrm{kg{\cdot}m}}{\mathrm{s}} \end{aligned}$$

Appendix A: Useful Numbers

Conversion Factors:

Length	cm	meter	km	in	ft	mi
1 cm =	1	10^{-2}	10^{-5}	0.3937	3.281×10^{-2}	6.214×10^{-6}
1 m =	100	1	10^{-3}	39.37	3.281	6.214×10^{-4}
1 km =	10^5	1000	1	3.937×10^4	3281	06214
1 in =	2.540	2.540×10^{-2}	2.540×10^{-5}	1	8.333×10^{-2}	1.578×10^{-5}
1 ft =	30.48	0.3048	3.048×10^{-4}	12	1	1.894×10^{-4}
1 mi =	1.609×10^5	1609	1.609	6.336×10^4	5280	1

Mass	g	kg	slug	u
1 g =	1	0.001	6.852×10^{-2}	6.022×10^{26}
1 kg =	1000	1	6.852×10^{-5}	6.022×10^{23}
1 slug =	1.459×10^4	14.59	1	8.786×10^{27}
1 u =	1.661×10^{-24}	1.661×10^{-27}	1.138×10^{-28}	1

An object with a *weight* of 1 lb has a *mass* of 0.4536 kg.

Earth Data

Mass	5.97×10^{24} kg
Mean Radius	6.37×10^6 m
Orbital Period	3.16×10^7 s
Mean Distance from Sun	1.50×10^{11} m
Mean Density	5.5×10^3 kg/m^3
Surface gravity	$9.81 \frac{m}{s^2}$

Sun

Mass	1.99×10^{30} kg
Mean Radius	6.96×10^8 m
Mean Distance from Galactic Center	2.6×10^{20} m
Mean Density	1.4×10^3 kg/m^3
Surface Temperature	5.8×10^3 K