Worked Examples from Introductory Physics
Vol. V: Electric Currents and Magnetic Fields

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Contents

To the Student. Yeah, *You.*

Hi. It's me again. Since you have obviously read all the stuffy pronouncements about the purpose of this problem–solving guide before, I won't make them again here.

I will point out that I've got *lots* more work to do on Volume 5, and I'm just making it available so that these chapters (such as they are) may be of some help to you. In fact, the whole set of books is a perpetual work in progress.

However....

Reactions, please!

Please help me with this project: Give me your reaction to this work: Tell me what you liked, what was particularly effective, what was particularly confusing, what you'd like to see more of or less of. I can be reached at `murdock@tntech.edu` or even at x–3044. If this effort is helping you to learn physics, I'll do more of it!

DPM

Chapter 1

Electric Current and Resistance

1.1 The Important Stuff

1.1.1 Electric Current

The topics covered up to now in your physics course have dealt with concentrations of electric charge which stay in one place, i.e. *electrostatics*. We now deal with the consequences of charge *moving* through a conductor, and the consequences are numerous, interesting and quite useful for modern technology. We now will work with **electrodynamics**.

In particular, we will study the flow of charge through **electric circuits**, i.e. networks of conductors, which might look like the one shown in Fig. 1.1.

In physics, we restrict this study to simple networks, focussing on the physical principles involved; for complex networks involving exotic types of conductors, consult your local engineering department!

Charges moves through the different parts of a circuit; it is important to measure the rate at which charge moves. Since charge cannot accumulate to any extent in the parts of a circuit, if we look at any one branch of a circuit the same net number of charges will be crossing a cross-sectional area per unit time, as shown in Fig. 1.2. Note that in this figure I show *positive* charges doing the actual motion, which is not really the case for normal conductors; it is the negatively charged *electrons* which are in motion. However for all of our

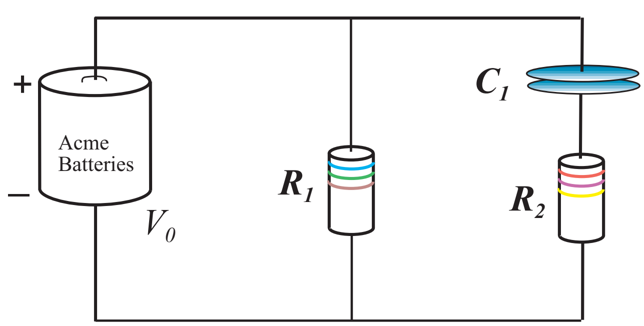

Figure 1.1: Electric circuits!

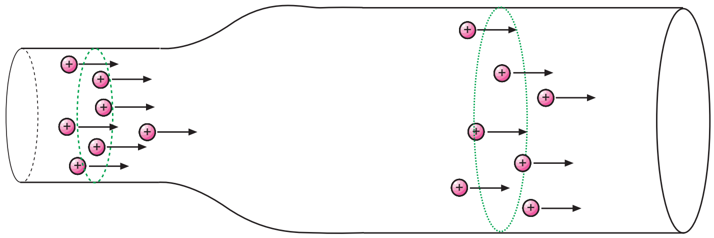

Figure 1.2: Current is the same in the skinny part of the wire as in the fat part; there are the same number of charges crossing a cross-sectional area per time.

work it will not make any difference if we pretend that an equal number of positive charge are moving in the direction *opposite* to that of the electron motion. We use this convention because it was the one in use before it was known that negatively–charged electrons were the ones that were in motion.

Suppose in a given branch of a circuit a small amount of charge dq passes through a given cross–sectional area in a time dt. Then the **electric current** in that branch is given by

$$I = \frac{dq}{dt} \tag{1.1}$$

Of course, if the current is constant the we can use $I = \frac{\Delta q}{\Delta t}$.

From its definition in Eq. 1.1, electric current must have SI units of coulombs per second ($\frac{\mathrm{C}}{\mathrm{s}}$) which is called an **ampere**[1]. Thus:

$$1\,\text{ampere} = 1\,\mathrm{A} = 1\,\frac{\mathrm{C}}{\mathrm{s}}\ .$$

Actually, we've seen this unit before (in Chapter 1) when we introduced the coulomb because it is actually the ampere which is easier to measure and standardize.

The definition of current involves a summation of the rates of passage of charges over the entire cross-sectional area, such as those in Fig. 1.2. Over any one tiny bit of this surface, if the density (number per volume) of charge carriers (with charge q) is n, and their average speed is v_d (the **drift velocity**), then through some small area dA we get an electric current dI given by

$$I = nqv_dA\ . \tag{1.2}$$

It is also useful to define a **current density** in the circuit branch. If the flow of charges is uniform over the cross–sectional area, then the current density is

$$J = \frac{I}{A} \tag{1.3}$$

which has units of $\frac{\mathrm{A}}{\mathrm{m}^2}$.

Comparing Eq. 1.3 with Eq. 1.2 (and generalizing it to a *vector* equation) gives

$$\mathbf{J} = nq\mathbf{v}_d \tag{1.4}$$

so that the current density vector $\mathbf{J}$ points in the same direction as the mean drift velocity $\mathbf{v}_d$.

[1]Named in honor of the. . . uh. . . English physicist Jim Ampere (c.1835–1779) who did some electrical experiments in. . . um. . . Heidelberg. That's it, Heidelberg.

1.1.2 Resistance & Ohm's Law

Current flows in a conductor when an electric potential is applied across its ends. What is the relation between the amount of current flowing (I) and the potential difference ("voltage", V)? Empirically, it is found that for "normal" materials, the current through a conductor is proportional the potential difference applied across its ends: $I \propto V$. The ratio of voltage to current is the **resistance** of the conductor:

$$R = \frac{V}{I} \tag{1.5}$$

which is the same as:

$$V = IR \tag{1.6}$$

and is known as **Ohm's Law**.

Resistance is a scalar quantity and from Eq. 1.5 has units of $\frac{\text{Volt}}{\text{Coul}}$, which is called an **ohm**[2]. Thus:

$$1\,\text{ohm} = 1\,\Omega = 1\,\frac{\text{volt}}{\text{ampere}}$$

The resistance of a particular conductor depends on the material of which it is made and its size and shape. For a simple shape like a cylinder (with electrical leads attached to the ends), we have a simple expression for the resistance. It is

$$R = \rho\frac{L}{A} \tag{1.7}$$

where L is the length of the conductor and A is its cross-sectional area. The proportionality constant ρ depends on the material from which the conductor is made. From its definition we see that ρ must have units of $\Omega \cdot \text{m}$. For example,

$$\begin{aligned} \rho_{\text{Copper}} &= 1.69 \times 10^{-8}\,\Omega \cdot \text{m} \\ \rho_{\text{Aluminum}} &= 2.75 \times 10^{-8}\,\Omega \cdot \text{m} \end{aligned}$$

Actually, the real definition of resistivity comes from a more general version of Ohm's Law which relates the electric field (vector) within the conductor to the current density (vector):

$$\mathbf{E} = \rho\mathbf{J} \tag{1.8}$$

The resistance of a certain conductor can also depend on its temperature. While this dependence might not be important for some conductors it can be important for those cases where a conductor is (intentionally!) heated to very large temperatures. Empirically it is found that a linear dependence of R on T (Celsius or Kelvin; it doesn't matter for a general linear relation) works pretty well, and we use a relation of the form:

$$\rho - \rho_0 = \rho_0\alpha(T - T_0) \qquad R - R_0 = R_0\alpha(T - T_0) \tag{1.9}$$

[2]Named in honor of the... uh... Polish physicist Jim Ohm (1215–1492) who did some electrical experiments in... um... Prague. That's it, Prague.

1.1.3 Dissipated Power

As an amount of charge dq passes through a resistor it "sees" a change in electrical potential IR and thus a change in electrical potential energy $(IR)dq$. The energy lost by the charge goes into the thermal energy of the resistor since energy is never really lost.

If dt is the time over which this charge dq has passed through the resistor then the *rate* at which energy is being "lost" in the resistor (the dissipated power, P) is

$$P = \frac{IR\,dq}{dt} = IR(I) = I^2R$$

where we used $\frac{dq}{dt} = I$. Since IR is the voltage drop across the resistor, we can also write this as $P = VI$. Thus we have:

$$P = IV = I^2R = \frac{V^2}{R} \tag{1.10}$$

We have already encountered the quantity power in the mechanics part of these notes. There is was defined as work done per time (energy per time, same as here) and the SI unit of power was given as $\frac{\mathrm{J}}{\mathrm{s}}$, also known as the **watt**[3]

1.2 Worked Examples

1.2.1 Electric Current

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1. A current of $5.0\,\mathrm{A}$ exists in a $10\,\Omega$ resistor for $4.0\,\mathrm{min}$. How many (a) coulombs and (b) electrons pass through any cross section of the resistor in this time?

(a) Using the constant–current version of Eq. 1.1, (namely $I = \frac{\Delta q}{\Delta t}$), for the given current I and time interval Δt we get:

$$\begin{aligned} \Delta q &= I\Delta t = (5.0\,\mathrm{A})(4.0\,\mathrm{min})\left(\frac{60\,\mathrm{s}}{1\,\mathrm{min}}\right) \\ &= 1.2 \times 10^3\,\mathrm{C} \end{aligned}$$

So 1.2×10^3 coulombs pass through any cross section of the wire in $4.0\,\mathrm{min}$.

(b) In part (a) we have found the absolute value of the electric charge passing through any part of the wire; to find the number of *electrons* which make up this charge, divide this by the absolute value of the electron charge e:

$$N = \frac{\Delta q}{e} = \frac{(1.2 \times 10^3\,\mathrm{C})}{(1.602 \times 10^{-23}\,\mathrm{C})} = 7.5 \times 10^{21}\,\mathrm{electrons}$$

[3]Named in honor of the...uh...Scottish physicist Jim Watt (1736–1819) who did some mechanical experiments in...um...Glasgow. That's it, Glasgow.

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2. In a particular cathode ray tube, the measured beam current is $30\,\mu$A. How many electrons strike the tube screen every $40\,$s?

Here, I is constant (and as used here it gives the *magnitude* of the flow of charge) so the amount of charge which strikes the screen has a magnitude

$$\Delta q = I\Delta t = (30\times 10^{-6}\,\mathrm{A})(40\,\mathrm{s}) = 1.2\times 10^{-3}\,\mathrm{C} \ ,$$

that is, a total charge of $-1.2\times 10^{-3}\,$C (since it is *electrons* which move in the tube) hits the screen. Use the charge per electron to find the number of electrons:

$$N = (-1.2\times 10^{-3}\,\mathrm{C})\left(\frac{1\,\mathrm{electron}}{-1.60\times 10^{19}\,\mathrm{C}}\right) = 7.5\times 10^{15}\,\mathrm{electrons}$$

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3. Suppose that the current through a conductor decreases exponentially with time according to

$$I(t) = I_0 e^{-t/\tau}$$

where I_0 is the initial current (at $t=0$), and τ is a constant having dimensions of time. Consider a fixed observation point within the conductor. (a) How much charge passes this point between $t=0$ and $t=\tau$? (b) How much charge passes this point between $t=0$ and $t=10\tau$? (c) How much charge passes this point between $t=0$ and $t=\infty$?

(a) The relation between charge increment dq and time increment dt can be written using Eq. 1.1 as:

$$dq = idt = I_0 e^{-t/\tau} dt \ .$$

To sum up all the charge passing a point between $t=0$ and $t=\tau$, do the integral on time to get:

$$\begin{aligned} q &= \int_0^\tau i(t)\,dt = \int_0^\tau I_0 e^{-t/\tau}\,dt \\ &= -I_0\tau e^{-t/\tau}\Big|_0^\tau = -I_0\tau\left(e^{-1}-1\right) \\ &= I_0\tau(1-e^{-1}) = (0.623)I_0\tau \end{aligned}$$

(b) For the charge passing the observation point between $t=0$ and $t=10\tau$ do the same integral as in (a) but from 0 to 10τ:

$$\begin{aligned} q &= \int_0^{10\tau} I_0 e^{-t/\tau}\,dt \\ &= -I_0\tau e^{-t/\tau}\Big|_0^{10\tau} = -I_0\tau\left(e^{-10}-1\right) \\ &= I_0\tau(1-e^{-10}) = (0.9995)I_0\tau \end{aligned}$$

(c) Now do the same integral, but from 0 to ∞:

$$\begin{aligned} q &= \int_0^\infty I_0 e^{-t/\tau}\, dt \\ &= -I_0\tau e^{-t/\tau}\Big|_0^\infty = -I_0\tau\,(0-1) \\ &= I_0\tau = I_0\tau \end{aligned}$$

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4. A Van de Graaff generator produces a beam of $2.0-\ \mathrm{MeV}$ deuterons, which are heavy hydrogen nuclei containing a proton and a neutron. (a) If the beam current is $10.0\,\mu\mathrm{A}$, how far apart are the deuterons in the beam? (b) Is their electrostatic repulsion a factor of beam stability? Explain. [The deuteron mass is $3.343\times10^{-27}\,\mathrm{kg}$.]

(a) In a beam of deuterons coming from an accelerator, the particles are definitely *not* equally spaced; but we will pretend that they are so that we will find an *average* spacing between the deuterons in the beam.

Note that the charge of a deuteron is the same as that of the proton, namely $q_{\mathrm{deut}} = +e$.

So imagining that we have such an orderly beam, we can use the defining relation for current, $I = \Delta q/\Delta t$ (i.e. a charge Δq passes by any given point in a time Δt) to solve for the time it takes a charge $+e$ to pass by. This is:

$$\Delta t = \frac{\Delta q}{I} = \frac{(1.602\times10^{-19}\,\mathrm{C})}{(10.0\times10^{-6}\,\mathrm{A})} = 1.60\times10^{-14}\,\mathrm{s}$$

We can get the distance between the deuterons if we know the *speed* of the particles. We are given their kinetic energy:

$$K = \tfrac{1}{2}mv^2 = (2.0\times10^6\,\mathrm{eV})\left(\frac{1.602\times10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}}\right) = 3.20\times10^{-14}\,\mathrm{J}$$

from which we get the speed:

$$v^2 = \frac{2K}{m} = \frac{2(3.20\times10^{-14}\,\mathrm{J})}{(3.343\times10^{-27}\,\mathrm{kg})} = 1.91\times10^{14}\,\tfrac{\mathrm{m}^2}{\mathrm{s}^2}$$

so that $v = 1.38\times10^7\,\frac{\mathrm{m}}{\mathrm{s}}$.

The (uniform) separation of the deuterons is just the distance that one deuteron travels in the time Δt found above, so that

$$\Delta x = v\Delta t = (1.38\times10^7\,\tfrac{\mathrm{m}}{\mathrm{s}})(1.60\times10^{14}\,\mathrm{s}) = 2.22\times10^{-7}\,\mathrm{m}$$

(b) To tell if electrostatic repulsion between the ions is important for the stability of the beam, we can think about comparing the ions moving in the beam to ions moving freely, i.e. far away from any other charges.

The potential energy of a pair of deuterons separated by the distance found above (as compared with their energy at infinite separation) is

$$\begin{aligned} U_{\rm elec} &= k\frac{e^2}{\Delta x} = \left(8.99\times 10^{9}\,\tfrac{\rm N\cdot m^2}{\rm C^2}\right)\frac{(1.60\times 10^{-19}\,{\rm C})^2}{(2.22\times 10^{-7}\,{\rm m})} \\ &= 1.04\times 10^{-21}\,{\rm J} = 6.5\times 10^{-3}\,{\rm eV} \end{aligned}$$

which is much smaller than the kinetic energy of the particles, which is of the order of MeV's. We can guess from this that the motion of the deuterons in this beam is not much different from that of free particles and the stability of beam is not influenced (much) by the mutual repulsion of the ions.

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5. Calculate the average drift speed of electrons travelling through a copper wire with a cross-sectional area of $1.00\,{\rm mm}^2$ when carrying a current of $1.00\,{\rm A}$ (values similar to those for he electric wire to your study lamp). It is known that about one electron per atom of copper contributes to the current. The atomic weight of copper is 63.54, and its density is $8.92\,{\rm g/cm^3}$

Eventually, we'll get $v_{\rm d}$ from Eq. 1.2, $I = nqv_dA$, so first we find n, the number density of freely-moving electrons in the wire. Now the *mass* density of copper is

$$\rho = 8.92\,\tfrac{\rm g}{\rm cm^3} = 8.92\times 10^{3}\,\tfrac{\rm kg}{\rm m^3}$$

and using the atomic weight (0.06354 kg for each mole) and Avogadro's number we can get the *number* density of coper atoms:

$$\begin{aligned} n_{\rm Cu} &= 8.92\times 10^{3}\,\tfrac{\rm kg}{\rm m^3}\left(\frac{1\,{\rm mole}}{0.06354\,{\rm kg}}\right)\left(\frac{6.022\times 10^{23}\,{\rm atoms}}{\rm mole}\right) \\ &= 8.454\times 10^{28}\,\tfrac{\rm atom}{\rm m^3} \end{aligned}$$

Now since there is only *one* electron for each copper atom that contributes to the current i.e. is freely–moving), then the number density of the conduction electrons is also $n = 8.454\times 10^{28}\,{\rm m^{-3}}$.

Now use Eq. 1.2 to get the drift speed. Assume that the total current $1.00\,{\rm A} = 1.00\,\frac{\rm C}{\rm s}$ is uniform over the cross-sectional area, and use the magnitude of the charge of the carriers $q = e = 1.60\times 10^{-19}\,{\rm C}$ and get:

$$\begin{aligned} v_{\rm d} &= \frac{I}{nqA} \\ &= \frac{(1.00\,\frac{\rm C}{\rm s})}{(8.454\times 10^{28}\,{\rm m^{-3}})(1.60\times 10^{-19}\,{\rm C})(1.00\times 10^{-6}\,{\rm m^3})} = 7.4\times 10^{-5}\,\tfrac{\rm m}{\rm s} \end{aligned}$$

The drift speed of the electrons is $7.4\times 10^{-5}\,\frac{\rm m}{\rm s}\ldots$ a rather small speed!

1.2.2 Resistance & Ohm's Law

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6. A conducting wire has a $1.0\,\mathrm{mm}$ diameter, a $2.0\,\mathrm{m}$ length, and a $50\,\mathrm{m\Omega}$ resistance. What is the resistivity of the material?

The cross-sectional area of the wire is

$$A = \pi r^2 = \pi \frac{d^2}{4} = \pi \frac{(1.0 \times 10^{-3}\,\mathrm{m})^2}{4} = 7.85 \times 10^{-7}\,\mathrm{m}^2$$

Then from Eq. 1.7 the resistivity ρ is given by

$$\begin{aligned} \rho &= \frac{RA}{L} \\ &= \frac{(50 \times 10^{-3}\,\Omega)(7.85 \times 10^{-7}\,\mathrm{m}^2)}{(2.0\,\mathrm{m})} = 2.0 \times 10^{-8}\,\Omega \cdot \mathrm{m} \end{aligned}$$

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7. A steel trolley-car wire has a cross-sectional area of $56.0\,\mathrm{cm}^2$. What is the resistance of $10.0\,\mathrm{km}$ of rail? The resistivity of the steel is $3.00 \times 10^{-7}\,\Omega \cdot \mathrm{m}$.

We have the resistivity ρ, cross-sectional area and length. (Note that $1\,\mathrm{cm}^2 = 10^{-4}\,\mathrm{m}^2$!) Then Eq. 1.7 gives the resistance of the rail:

$$R = \rho \frac{L}{A} = (3.00 \times 10^{-7}\,\Omega \cdot \mathrm{m}) \frac{(10.0 \times 10^2\,\mathrm{m})}{(56.0 \times 10^{-4}\,\mathrm{m}^2)} = 0.536\,\Omega$$

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8. When $115\,\mathrm{V}$ is applied across a wire that is $10\,\mathrm{m}$ long and has a $0.30\,\mathrm{mm}$ radius, the current density is $1.4 \times 10^4\,\mathrm{A/m}^2$. Find the resistivity of the wire.

By doing some algebra first we can save ourselves some button-pushing. Write out Ohm's law and substitute for the resistance R:

$$\begin{aligned} V &= IR = I\left(\rho \frac{L}{A}\right) \\ &= \left(\frac{I}{A}\right) \rho L = J \rho L \end{aligned}$$

where we've also used the definition of the current density, $J = I/A$, since we are given its value. Solve for ρ and plug in the numbers:

$$\rho = \frac{V}{JL} = \frac{(115\,\mathrm{V})}{(1.4 \times 10^4\,\mathrm{A/m}^2)(10\,\mathrm{m})} = 8.2 \times 10^{-4}\,\Omega \cdot \mathrm{m}$$

So the resistivity of the wire is $8.2 \times 10^{-4}\,\Omega \cdot \mathrm{m}$.

As it turns out, we did not need to know the radius of the wire.

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9. A common flashlight bulb is rated at $0.30\,\mathrm{A}$ and $2.9\,\mathrm{V}$ (the values of the current and voltage under operating conditions). If the resistance of the bulb filament at room temperature ($20°\mathrm{C}$) is $1.1\,\Omega$, what is the temperature of the filament when the bulb is on? The filament is made of tungsten, for which $\alpha = 4.5 \times 10^{-3}/\mathrm{K}$.

From Ohm's law we can find the resistance of the bulb at the temperature of the normal operating conditions (hot!):

$$R = \frac{V}{I} = \frac{(2.9\,\mathrm{V})}{(0.30\,\mathrm{A})} = 9.7\,\Omega$$

This is in contrast to the resistance of the filament at room temperature, $1.1\,\Omega$.

Equation 1.9 gives the dependence of resistivity (ρ) on temperature. The same relation holds true for the *resistance* R of a circuit element because to get R from ρ we multiply by L/A (assume a cylindrical conductor). So we use

$$R - R_0 = R_0\alpha(T - T_0)$$

with our values for $T_0 = 20°\mathrm{C}$ and solve for T. We get:

$$(T - T_0) \;=\; \frac{1}{\alpha}\frac{(R - R_0)}{R_0} \;=\; \frac{1}{(4.5 \times 10^{-3}/\mathrm{K})}\frac{(9.7\,\Omega - 1.1\,\Omega)}{1.1\,\Omega} \;=\; 1.7 \times 10^3\,\mathrm{K} \;=\; 1.7 \times 10^3\,\mathrm{C}°$$

Then the operating temperature T is

$$T \;=\; T_0 + 1.7 \times 10^3\,\mathrm{C}° \;=\; 20°\mathrm{C} + 1.7 \times 10^3\,\mathrm{C}° \;=\; 1.7 \times 10^3\,°\mathrm{C}$$

1.2.3 Dissipated Power

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10. A certain x-ray tube operates at a current of $7.0\,[\mathrm{mA}$ and a potential difference of $80\,\mathrm{kV}$. What is its power in watts?

The problem gives us the current I passing through the tube and the potential difference (voltage drop) V across the tube. Then Eq. 1.10 gives us the power dissipated:

$$P \;=\; IV \;=\; (7.0 \times 10^{-3}\,\mathrm{A})(80 \times 10^3\,\mathrm{V}) \;=\; 5.6 \times 10^2\,\mathrm{W}$$

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11. Thermal energy is produced in a resistor at a rate of $100\,\mathrm{W}$ when the current is $3.00\,\mathrm{A}$. What is the resistance?

Here we are given the power dissipated in the resistor and the current passing through it. Then we can get the resistance by using $P = I^2R$. This gives:

$$R \;=\; \frac{P}{I^2} \;=\; \frac{(100\,\mathrm{W})}{(3.00\,\mathrm{A})^2} \;=\; 11.1\,\Omega$$

Chapter 2

DC Circuits

2.1 The Important Stuff

2.1.1 Analyzing Circuits

We now extend the ideas of the last chapter to situations where circuit elements (such as batteries and resistors) are attached together in more complicated (and interesting!) ways. We will look at circuits with multiple loops and calculate the important quantities for these circuits: The potential differences between various points in the circuit and the currents flowing in the different branches of the circuit. For example, we might be given a circuit, part of which may look like the one shown in Fig. 2.1. Given the values of V and $R_1, R_2, \ldots$ it would be our job to find the currents in the different branches, $i_1, i_2 \ldots$ and perhaps the potential difference between the points a and b.

There is an important point to be made about the direction of the arrow we put in our pictures and the *actual* direction in which current is flowing: For the sake of getting ourselves straight, we might *begin* a circuit problem by writing down arrows to reference the directions of the currents as in Fig. 2.1. But we may later find that one of the i's is negative! That's not a problem; it only means that we guessed wrong for the direction of the current in one

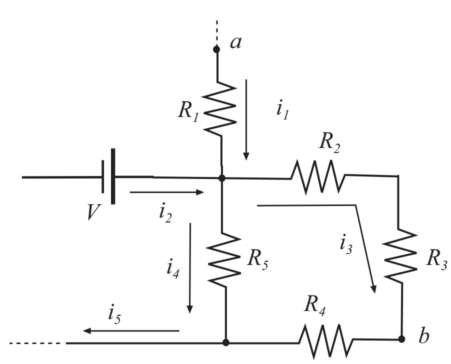

Figure 2.1: Analyzing a circuit: What are the currents $i_1, i_2, \ldots$? What is the potential difference between the points a and b ?

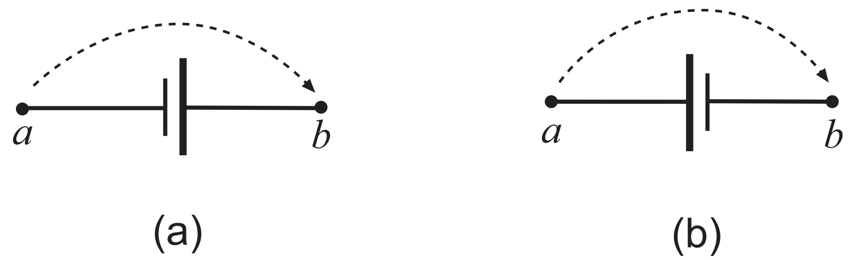

Figure 2.2: Hopping over an emf source $\mathcal{E}$. (The positive terminal of the source is represented by the thick bar.) (a) Going from the negative end to the positive end the change in potential is $\mathcal{E}$. (b) Going from the positive end to the negative end the change in potential is $-\mathcal{E}$.

branch. It's the same thing as drawing the arrow in the opposite direction and then getting a positive result for that i. There is no problem as long as we watch the directions and stay consistent.

2.1.2 Analyzing Circuits

Our circuits will contain various conducting elements connected by wires whose lengths and orientations we can ignore. The elements of these circuits are:

• **Batteries; emf Devices**

These include chemical batteries and constant–voltage sources (often called "power supplies") which run from wall outlets. When a current I flows through such a device, the energy of the charges increases at a rate

$$P = \mathcal{E}\frac{dq}{dt} = \mathcal{E}i$$

When, in our our analysis of a circuit we hop from the negative terminal of a battery to the positive terminal, the potential difference is $+\mathcal{E}$, where $\mathcal{E}$ is the rated voltage of the device. If we have occasion to hop from the positive to the negative terminal (and we will) there is a change in potential of $-\mathcal{E}$. This is shown in Fig. 2.2.

Actually, when we figure an emf device into our circuits we are really talking about an **ideal emf device**. In reality, when current I flows through an emf device there is an increase in potential $\mathcal{E}$ but also a *drop* in potential of magnitude ir, where r is the **internal resistance** of the (real) emf device.

• **Resistors**

The change in potential from one end to the other of a resistor R depends on the current i; by Ohm's Law, $|\mathcal{E}| = |iR|$. But we need to watch for *directions* of the currents and use the right signs when we go hopping around the circuit finding our changes in potential.

When we go from one end of a resistor R to the other end in the *same direction* as the assigned current i, there is a change in potential $-iR$. When we go from one end to the other *against* the direction of the assigned current i there is a change in potential $+iR$. This is illustrated in Fig. 2.3.

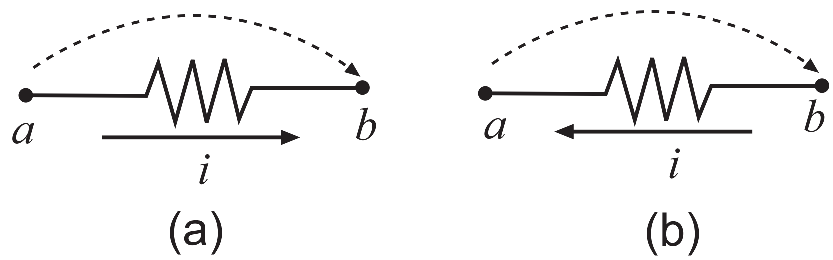

Figure 2.3: Hopping over a resistor R. (a) Going from a to b *along with* the assigned direction of the current, the change in potential is $-iR$. (b) Going from a to b *against* the assigned direction of the current, the change in potential is $+iR$. Note, we are *not* saying that i is positive here.

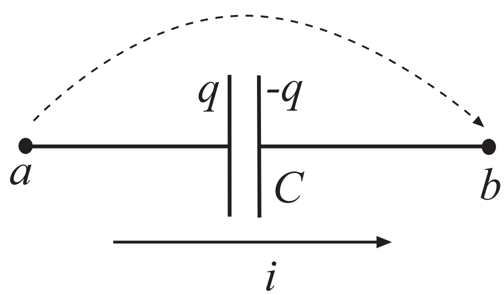

Figure 2.4: Hopping over a capacitor C in the analysis of a circuit. Going from a to b with the charge of the capacitor q defined as shown, there is a change in potential $-\frac{q}{C}$. Charge q is related to current i by $i = \frac{dq}{dt}$.

• **Capacitors**

We will encounter capacitors again when we discuss circuits where we switch on the contact between a capacitor's plates and a battery (with a resistance in the circuit); we will study how the current into the plates (and the charge on the plates) changes with time. Recall that from our old formula $q = CV$ (that is, $V = q/C$) when we go from the plate with charge q to the plate with charge $-q$, the change in potential is $-q/C$, as illustrated in Fig. 2.4.

• **Inductors**

Whoa! We haven't encountered these yet, and for the time being we won't put them in our circuits! We need to study magnetic fields to understand what an inductor does!

However, for the record, an inductor is a coil of wire, the potential drop across a inductor depends on the *rate of change* of the current and is given by $+L\frac{di}{dt}$. L is the measure of the property called "self–inductance" and is measured in Henrys. But we'll get to that later on.

We use these rules for finding potential differences along with two facts:

• The sum of the potential differences taken around any closed loop in a circuit must be zero.

• The sum of the currents *entering* any junction must equal the sum of the currents *exiting* that junction. Then we can find all the currents if we just work hard enough.

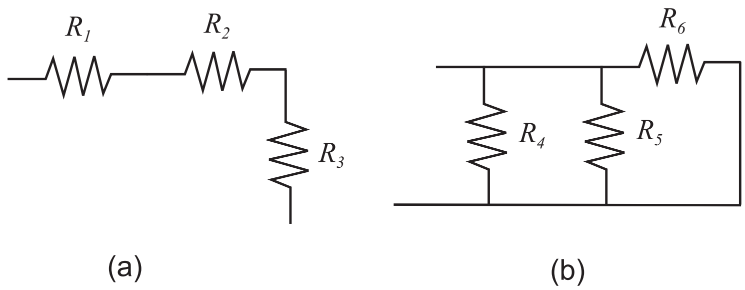

Figure 2.5: (a) Resistors R_1, R_2 and R_3 are connected in series. (b) Resistors R_4, R_5 and R_6 are connected in parallel.

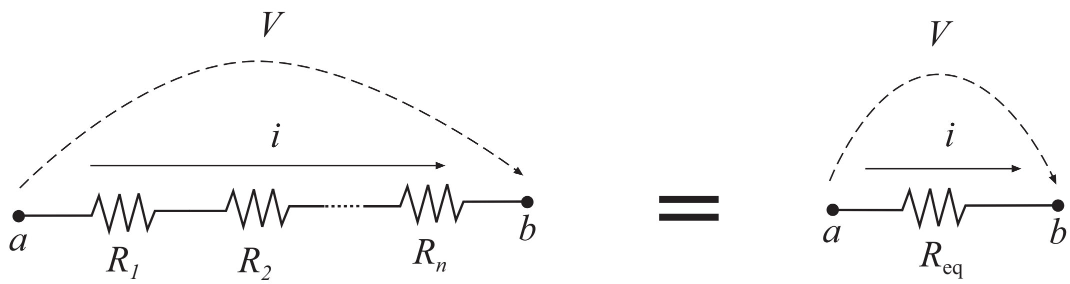

Figure 2.6: A set of resistors R_1, R_2, $\ldots R_n$ connected in series has a current i common to all the resistors and a potential drop V across the whole set. For the purposes of finding the current i, it can be treated as a single equivalent resistor given by $R_{\rm eq} = R_1 + R_2 + \ldots + R_n$. Then $V = iR_{\rm eq}$.

These rules are known as the **Kirchhoff loop rule** and the **Kirchhoff junction rule**, respectively. It is necessary to use them when we have to analyze a circuit with multiple loops.

2.1.3 Resistors in Series and in Parallel

Oftentimes in a circuit we have a set of resistors joined together either by connecting them end-to-end (a **series** connection) or by connecting *both* of their ends together (a **parallel** connection). The two types of connections are illustrated in Fig. 2.5.

When we have resistor combinations like these, we can take some shortcuts in our analysis of the circuit; we can treat the set as a *single* equivalent resistor, at least as far as the current entering the set is concerned.

• **Series Combination**

Suppose we have a set of resistors in a string, with values R_1, R_2, $...R_n$; the resistors are joined end–to–end with *no other connections* made between the resistors in the string. Then the resistors can be replaced the equivalent resistance $R_{\rm eq}$ given by

$$R_{\rm eq} = R_1 + R_2 + \ldots R_n \qquad (2.1)$$

This substitution in diagrammed in Fig. 2.6.

• **Parallel Combination**

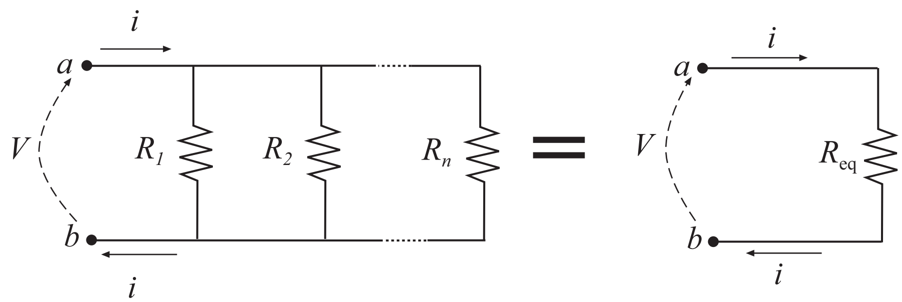

Figure 2.7: A set of resistors R_1, R_2, $\ldots R_n$ connected in parallel has a potential difference V common to all the resistors. A *total* current i enters and exits the combination. For the purposes of finding this current it can be treated as a single equivalent resistor given by $R_{\rm eq}^{-1} = R_1^{-1} + R_2^{-1} + \ldots + R_n^{-1}$. Then $V = iR_{\rm eq}$.

Suppose we have a set of resistors with values R_1, R_2, $...R_n$; the ends of these resistors are joined together. Then the resistors can be replaced the equivalent resistance $R_{\rm eq}$ given by

$$\frac{1}{R_{\rm eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots \frac{1}{R_n} \tag{2.2}$$

This substitution in diagrammed in Fig. 2.7.

In the special case where we have only two resistors R_1 and R_2 in parallel, the formula gives:

$$\frac{1}{R_{\rm eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

so then

$$R_{\rm eq} = \frac{R_1 R_2}{(R_1 + R_2)} \tag{2.3}$$

2.1.4 Solving Big Messy DC Circuits

Big messy DC circuits can be solved by applying the Kirchhoff rules to get a set of *independent* linear equations for the unknown currents and then solving them for the currents in the separate branches.

• In each branch of the circuits assign a current, including a direction; draw a labelled arrow on the diagram for each current.

•

•

•

•

2.1.5 RC Circuits.

Now we have a look at a circuit in which the current is *not* constant. (More will follow!) We consider the circuit shown in Fig. 2.8. The circuit has a resistor R and a capacitor C

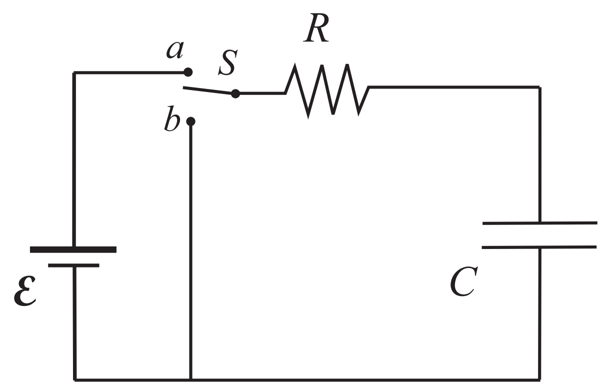

Figure 2.8: *RC* circuit; with the switch thrown to position *a* the capacitor *C* builds up a charge. Then with the switch thrown to *b* the capacitor discharges.

connected in series. Depending on whether the switch in thrown to a ro b we can include or not include a battery with emf $\mathcal{E}$ is series with them.

Suppose the capacitor is initially uncharged and at time $t = 0$ the switch is thrown to a. One can show that the charge on the capacitor is given by

$$q(t) = C\mathcal{E}(1 - e^{-t/(RC)}) \tag{2.4}$$

and the current in the circuit is

$$i(t) = \frac{dq}{dt} = \left(\frac{\mathcal{E}}{R}\right) e^{-t/(RC)} \tag{2.5}$$

The combination RC that occurs in both of these results has units of *time*. We represent it by the symbol τ:

$$\tau = RC \quad . \tag{2.6}$$

τ is called the **capacitive time constant** for the circuit.

Eq. 2.4 tells us that at very "large" values of t the exponential term is very small and so the charge q is very nearly equal to $C\mathcal{E}$. This is the "full", or equilibrium value of the charge on the capacitor. The value of q at a time $t = \tau$ after we close the switch turns out to be about 0.63 (i.e. 63%) of this value, so that τ gives a measure of the time required for "most" of the equilibrium charge to collect.

Eq. 2.5 tells us that the current starts off with the value $\frac{\mathcal{E}}{R}$ (the value it would have if there were no capacitor in the circuit) and it falls off to zero at large values of t. At $t = \tau$ the current has decreased to about 37% of its initial value.

Now when we throw the switch to position b (at $t = 0$) the capacitor will lose its charge as the current goes the opposite way through the resistor (this time bypassing the battery). One can show that the charge on the capacitor is given by

$$q(t) = q_0 e^{-t/(RC)} \tag{2.7}$$

where q_0 was the charge on the capacitor when the switch was thrown to b. The current in the circuit is now:

$$i(t) = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right) e^{-t/(RC)} \tag{2.8}$$

(note, the minus sign tell us that the current flows the opposite way from the direction it had when C was charging).

2.2 Worked Examples

2.2.1 Analyzing Circuits

2.2.2 Resistors in Series and in Parallel

2.2.3 Solving Big Messy DC Circuits

2.2.4 RC Circuits.

Chapter 3

Magnetic Fields – Forces

3.1 The Important Stuff

3.1.1 Magnetic Fields

Just as a charge q experiences a force $\mathbf{F} = q\mathbf{E}$ from an electric field $\mathbf{E}$ (whether or not it is in motion), a *moving* charge q will experience a new kind of force if it is moving in a **magnetic field**.

The magnetic field is given the symbol $\mathbf{B}$, and like the electric field, it is a *vector* field, that is, at each point in space its value is specified by giving its three components (or by its direction and magnitude). As with electric fields, we can get a useful overall picture of a magnetic field by showing magnetic field lines; these give the direction of the $\mathbf{B}$ field at each point but not the magnitude.

The magnetic field $\mathbf{B}$ is measured in tesla; more on that in the next section.

3.1.2 Magnetic Force on a Moving Point Charge.

When a point charge q moves with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$, it experiences a force which is proportional to both the charge's speed and the magnitude of the field, but the *direction* of the force is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$. Furthermore, for a nonzero force, $\mathbf{B}$ must have a component perpendicular to $\mathbf{v}$; if the velocity is parallel to $\mathbf{B}$ there is no force.

If the angle between $\mathbf{v}$ and $\mathbf{B}$ is θ, then the magnitude of the magnetic force is

$$F = |q|vB\sin\theta$$

and the direction of the force is given by the right hand rule: Point your four fingers in the direction of $\mathbf{v}$ and let them sweep (bend) from $\mathbf{v}$ to $\mathbf{B}$. Then your thumb points in the direction of the force $\mathbf{F}$, *if q is positive*. If q is negative, $\mathbf{F}$ points in the opposite direction.

The force law given above is neatly expressed using the cross product:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \tag{3.1}$$

As mentioned above, the magnetic field is measured in **Tesla**[1].

3.1.3 Circular Motion of Particles in Magnetic Fields

If a particle enters a uniform magnetic field having a velocity perpendicular to the direction of the field it will undergo circular motion; the centripetal force is provided by the magnetic force.

Recall that a particle undergoes circular motion because there is a force *pulling it toward the center* of the circle. The centripetal force is perpendicular to the velocity and has magnitude mv^2/r where r is the radius of the circle. Here, the centripetal force is supplied by the magnetic force; since the velocity is perpendicular to the field, the magnetic force has magnitude qvB, where q is he absolute value of the charge of the particle. Equating the two expressions and cancelling a factor of v gives:

$$qB = \frac{mv}{r} \tag{3.2}$$

3.2 Worked Examples

[1]Named in honor of the…uh…Swedish physicist Jim Tesla (1935–2021) who did some electrical experiments in…um…Zürich. That's it, Zürich.

Appendix A: Useful Numbers

Conversion Factors

Length	**cm**	**meter**	**km**	**in**	**ft**	**mi**
1 cm =	1	10^{-2}	10^{-5}	0.3937	3.281×10^{-2}	6.214×10^{-6}
1 m =	100	1	10^{-3}	39.37	3.281	6.214×10^{-4}
1 km =	10^5	1000	1	3.937×10^4	3281	06214
1 in =	2.540	2.540×10^{-2}	2.540×10^{-5}	1	8.333×10^{-2}	1.578×10^{-5}
1 ft =	30.48	0.3048	3.048×10^{-4}	12	1	1.894×10^{-4}
1 mi =	1.609×10^5	1609	1.609	6.336×10^4	5280	1

Mass	**g**	**kg**	**slug**	**u**
1 g =	1	0.001	6.852×10^{-2}	6.022×10^{26}
1 kg =	1000	1	6.852×10^{-5}	6.022×10^{23}
1 slug =	1.459×10^4	14.59	1	8.786×10^{27}
1 u =	1.661×10^{-24}	1.661×10^{-27}	1.138×10^{-28}	1

An object with a *weight* of 1 lb has a *mass* of 0.4536 kg.

Constants:

$$
\begin{aligned}
e &= 1.6022 \times 10^{-19}\,\mathrm{C} = 4.8032 \times 10^{-10}\,\mathrm{esu} \\
\epsilon_0 &= 8.85419 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N{\cdot}m}^2} \\
k &= 1/(4\pi\epsilon_0) = 8.9876 \times 10^{9}\,\frac{\mathrm{N{\cdot}m}^2}{\mathrm{C}^2} \\
\mu_0 &= 4\pi \times 10^{-7}\,\frac{\mathrm{N}}{\mathrm{A}^2} = 1.2566 \times 10^{-6}\,\frac{\mathrm{N}}{\mathrm{A}^2} \\
m_{\mathrm{electron}} &= 9.1094 \times 10^{-31}\,\mathrm{kg} \\
m_{\mathrm{proton}} &= 1.6726 \times 10^{-27}\,\mathrm{kg} \\
c &= 2.9979 \times 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}} \\
N_A &= 6.0221 \times 10^{23}\,\mathrm{mol}^{-1}
\end{aligned}
$$