

Worked Examples from Introductory Physics
Vol. III: Thermal Physics

David Murdock

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Chapter 1

Temperature, Heat and Work

1.1 The Important Stuff

1.1.1 Thermal Physics

The next three chapters of this study guide deal with **thermal physics**. Here we finally deal with the fact that the objects and substances in the real world (solid, liquid or gas) are made of atoms and we have to consider the changes in the motions of the atoms if we are to have a truly complete description of a physical system.

Alas, atoms are tiny little beasties and in our usual laboratory physical systems there are *lots* of them, on the order of 10^{23} atoms. It is simply impossible to track the motion of all the atoms in a sample. But it is true that when an object gets “hotter” or “colder” the atoms *on average* will be moving faster or slower, and the difference can be *measured* because there will often be a change in the large-scale or **macroscopic** properties of the object. Examples of macroscopic properties are volume, pressure and **temperature**. We will study the macroscopic properties of a sample in order to have an understanding of the *overall* behavior of its atoms.

Whereas volume and pressure just involve familiar ideas about length and force, the idea of temperature requires some care so that it can be treated in a quantitative manner.

1.1.2 Temperature

When we put two objects “in contact”, the atoms in those objects can exchange energy. In doing so, some macroscopic (measurable) properties of the objects can change. If we wait long enough, those properties (actually, *all* the properties that one *could* measure) will be constant, and at that time we say that the objects are in **thermal equilibrium** with one another.

The so-called **Zereth Law of Thermodynamics** (there are three more!) states:

When object A is in thermal equilibrium with object B and object B is in thermal equilibrium with object C, then A will be in thermal equilibrium with C.

The Zereth Law tells us that there is *some* property that is common to objects in thermal

equilibrium. This property is **temperature**, and so the Zeroth Law is really telling us that temperature is a *meaningful* concept. Now we need a way to get a *quantitative* measure of temperature.

1.1.3 Measuring Temperature

Water (H₂O) has the property that at a specific temperature and pressure its three phases (solid liquid and gas) can exist in thermal equilibrium with each other. Clearly, this does not happen under “familiar” conditions; the pressure for this physical state is $P_3 = 4.58$ mm (very small!) and the temperature for this state (the **triple point** of water) is *defined* to be $T_3 = 273.16$ K. The “K” here is the unit of temperature known as the **Kelvin**. (Nowadays we say “Kelvins” and not “degrees Kelvin” when expressing a temperature.)

Good enough, but now how do we go about determining the temperature of some *other* physical system? The reasoning for getting other temperatures is somewhat tricky and involves a device, known as the **constant–volume gas thermometer**, shown in Fig. –. This device is intended to measure the temperature T of a “bath”. As we fill the bulb with a certain amount of some gas and put mercury in the lower part of the tube and then place the bulb in the bath.

The difference in pressure between the gas and the atmosphere results in a difference of heights h of the mercury in the two parts of the tube. (If h is positive then the gas is at lower pressure than the atmosphere.) If the gas pressure is P and the external pressure is P_0 then

$$P = P_0 - \rho gh$$

where ρ is the density of mercury.

On the assumption that the absolute temperature T is proportional to the gas pressure (which we will justify later on) then

$$T = CP$$

and in particular if the bulb of the constant–volume thermometer were put in contact with water at its triple point we would have

$$T_3 = CP_3 .$$

The constant C will depend on the type and amount of gas and the value of the fixed volume; together, these equations imply:

$$T = T_3 \left(\frac{P}{P_3} \right) = (273.16 \text{ K}) \left(\frac{P}{P_3} \right) \quad (1.1)$$

Using Eq. 1.1 one can measure the (absolute) temperature of any environment in which the gas bulb may be placed, but in fact Eq. 1.1 is not a very good prescription for T because the value may depend on the type, amount and volume of the gas used in the “thermometer”. But experience shows that when the amount (mass m) of gas in the bulb is *very very small* then Eq. 1.1 gives the same value for T , and so we can complete our hypothetical process of

temperature measurement by specifying that the mass of gas must be as small as is possible. With that in mind, the relation which defines the absolute temperature is

$$T = (273.16 \text{ K}) \left(\lim_{m \rightarrow 0} \frac{P}{P_3} \right) \quad (1.2)$$

1.1.4 Other Temperature Scales

In nearly every country of the world the temperature scale used in everyday life is the **Celsius** scale. It is related to the Kelvin scale by a constant *shift* so that the increments in temperature have the same size but the *zero* of the scale is different. The Celsius temperature T_C is given by:

$$T_C = T - 273.15^\circ \quad (1.3)$$

At a pressure of 1 atm, the freezing point of water (is about 0.00°C and the boiling point is about 100.0°C .

The Fahrenheit scale is used in everyday life in the US. The Fahrenheit temperature T_F is related to the Celsius temperature by:

$$T_F = \frac{9}{5}T_C + 32^\circ \quad (1.4)$$

At a pressure of 1 atm, the freezing point of water (is about 32°F and the boiling point is about 100.0°F .

1.1.5 Thermal Expansion

Solids typically undergo an increase in size as they increase in temperature. The fractional increase in any linear dimension of the solid is proportional to the change in temperature, so that for any dimension L , we have $\frac{\Delta L}{L} = \alpha \Delta T$, where α is a constant which characterizes the particular solid. Thus we have:

$$\Delta L = L\alpha\Delta T \quad (1.5)$$

For most problems of interest the change in any length of a solid is much smaller than the original length so that the “ L ” in Eq. 1.5 could really mean either the initial or final length.

Note that *any* linear dimension of the solid increases with increasing T . So if we heat up a sheet of metal with a circular hole, the size of the hole *also* increases.

When we consider liquids (which, being liquid, don’t have fixed shapes for which we can consider the increase in a linear dimension) we usually find that the volume occupied by liquid undergoes a fractional increase proportional to the change in temperature. Then, similar to Eq. 1.5 we have:

$$\Delta V = V\beta\Delta T \quad (1.6)$$

This equation is also applicable to solids; in that case the linear and volume coefficients of expansion are related by

$$\beta = 3\alpha \quad (1.7)$$

1.1.6 Temperature and the Absorption of Heat

As we have noted above, objects in thermal contact eventually come the same temperature. This occurs because **thermal energy** (the energy associated with the motions of the atoms in the objects) is transferred from the hot object to the cold object. This transfer of energy is known as **heat** and is represented by the symbol Q . Q is positive when the object in question *gains* thermal energy.

Heat has the same units as energy (in the SI system, joules). For historical reasons there is another unit commonly used for heat, the **calorie**. The relation between the two is:

$$1 \text{ calorie} = 4.186 \text{ joule} \quad (1.8)$$

The calorie is commonly used by chemists, and since many thermal physics problems are of a chemical nature we will be using it!

For many substances (such as the solids and liquids we discuss in this chapter) we can write down a simple relation between the heat transferred to an object and its change in temperature. Later we will have reason to think about the heat transferred to a gas, but that case is more complicated because a gas can (when allowed to) change its volume drastically and in doing so it exchanges energy with its surroundings by the process we classify as **work**. We will then have to worry about the *conditions* under which thermal energy is added to a gas. For solids and liquids we won't have to worry about this.

Oftentimes the heat added to a sample is proportional to its increase in temperature. If we call the constant of proportionality C , then

$$Q = C\Delta T \quad (1.9)$$

C is called the **heat capacity** of the object and it has (SI) units of J/K (joules per kelvin).

Now, if we take two pieces of some pure substance (say, iron) then in order to give the *same* change in temperature, we must transfer more heat to the larger sample. (We need to increase the energy of *more* atoms.) So the heat transferred is also proportional to the mass m of the sample. In this case it is more useful to write:

$$Q = cm\Delta T \quad (1.10)$$

where c now depends *only* on the *kind* of substance under consideration. The number c is called the specific heat of the substance and it has SI units of $\frac{\text{J}}{\text{kg}\cdot\text{K}}$, though it is often expressed in units of $\frac{\text{cal}}{\text{g}\cdot\text{K}}$. (Again, the latter units are used often in chemistry.)

The value of c for (liquid) water is especially simple in the chemists' units, and in fact it formerly provided the definition of the calorie:

$$C_{\text{water}} = 1.00 \frac{\text{cal}}{\text{g}\cdot\text{K}} = 4190 \frac{\text{J}}{\text{kg}\cdot\text{K}}$$

1.1.7 Heats of Transformation

An important case where a substance does *not* behave according to Eq. 1.10 is when the substance undergoes a **phase transition**. Familiar examples of changes in phase are the

transformation of liquid water to solid water “ice”) at 0.0°C and the transformation of solid CO_2 to gaseous CO_2 at -78.5°C (both at one atmosphere of pressure). In each case, the substance absorbs heat while remaining at the *same* temperature; as more heat is added, more of the substance is transformed.

A change of phase from solid to liquid is called melting (the reverse being freezing or fusing). The change from liquid to vapor is called vaporization (the reverse, condensation). The change from solid directly to vapor (as with CO_2) is called sublimation.

We consider the heat required to (say) melt a given quantity of some solid when the two phases are at the melting point temperature with no further change in temperature occurring. This amount of heat is proportional to the amount of the substance which melts. If we call the constant of proportionality L_f , then

$$Q = mL_f \quad (1.11)$$

L_f is called the **latent heat of fusion**; it has SI units of J/kg. (It can also be expressed in terms of *moles*, so e.g. chemists may express this quantity in terms of cal/mol.)

1.2 Worked Examples

1.2.1 Measuring Temperature; Temperature Scales

1. If your doctor tells you that your temperature is 310 kelvins above absolute zero, should you worry? Explain your answer.

Yes, you *should* worry... especially if you're spending your own money to see some wise-ass doctor who gives you your temperature in nerdy physics units. But let's trust the doctor and see if there's a reason to worry about your *health*.

All that I (DPM) know about health is that the body temperature of a healthy person should be close to 98.6°F . So what is 310 K in Fahrenheit degrees?

From Eq. 1.3 we find your temperature in Celsius degrees:

$$T_C = T - 273.15^\circ = (310.0 - 273.15)^\circ = 36.85^\circ\text{C}$$

Now use Eq. 1.4 to find the temperature in Fahrenheit degrees:

$$T_F = \frac{9}{5}T_C + 32^\circ = \frac{9}{5}(36.85^\circ) + 32^\circ = 98.3^\circ\text{F}$$

This seems to be close enough to 98.6°F that you *don't* need to worry about your health.

2. (a) In 1964, the temperature in the Siberian village of Oymyakon reached a value of -71°C . What temperature is this on the Fahrenheit scale? (b) The

highest officially recorded temperature in the continental United States was 134°F in Death Valley, California. What temperature is this on the Celsius scale?

(a) Here, $T_C = -71^\circ\text{C}$. Use Eq. 1.4 to get a Fahrenheit temperature from a Celsius one:

$$T_F = \frac{9}{5}T_C + 32^\circ = \frac{9}{5}(-71^\circ) + 32^\circ = -95.8^\circ\text{F}$$

(b) Here we want to get a Celsius temperature from one given in Fahrenheit degrees. Solving for T_F in Eq. 1.4, we get:

$$T_C = \frac{5}{9}(T_F - 32^\circ)$$

Then for $T_F = 134^\circ$,

$$T_C = \frac{5}{9}(134^\circ - 32^\circ) = 56.7^\circ\text{C}$$

1.2.2 Thermal Expansion

3. A steel rod has a length of exactly 20 cm at 30°C . How much longer is it at 50°C ? [Use $\alpha_{\text{Steel}} = 11 \times 10^{-6}/\text{C}^\circ$.]

The change in temperature of the steel rod is

$$\Delta T = 50^\circ\text{C} - 20^\circ\text{C} = 30^\circ\text{C}$$

and the length is 20.0 cm. Using the given value for the linear expansion coefficient α , we find the change in length from Eq. 1.5,

$$\Delta L = L\alpha\Delta T = (20.0\text{ cm})(11 \times 10^{-6}/\text{C}^\circ)(30^\circ\text{C}) = 6.6 \times 10^{-3}\text{ cm}$$

The length of the bar increases by $6.6 \times 10^{-3}\text{ cm}$.

4. An aluminum flagpole is 33 m high. By how much does its length increase as the temperature increases by 15°C ? [Use $\alpha_{\text{Al}} = 23 \times 10^{-6}/\text{C}^\circ$.]

Using the given value of α and Eq. 1.5, the change in length of the (aluminum) flagpole is

$$\Delta L = L\alpha\Delta T = (33\text{ m})(23 \times 10^{-6}/\text{C}^\circ)(15^\circ\text{C}) = 1.1 \times 10^{-2}\text{ m} = 1.1\text{ cm}$$

5. The Pyrex glass mirror in the telescope at the Mt. Palomar Observatory has a diameter of 200 in. The temperature ranges from -10°C to 50°C on Mt. Palomar. Determine the maximum change in the diameter of the mirror. [Use $\alpha_{\text{Pyrex}} = 3.2 \times 10^{-6}/\text{C}^\circ$.]

Recall that Eq. 1.5 gives the change in *any linear dimension* of the solid object, so in this case we can take L to be the diameter of the mirror. We can leave L in units of inches, in which case ΔL comes out in inches.

Considering a change in temperature of

$$\Delta T = 50^\circ\text{C} - (-10^\circ\text{C}) = 60^\circ\text{C}$$

the increase in the diameter is

$$\Delta L = L\alpha\Delta T = (200\text{ in})(3.2 \times 10^{-6}/\text{C}^\circ)(60^\circ\text{C}) = 3.8 \times 10^{-2}\text{ in}$$

The diameter of the mirror increases by 3.8×10^{-2} inches.

6. Soon after the Earth was formed, heat released by the decay of radioactive elements raised the average internal temperature from 100 to 3000 K, at about which value it remains today. Assuming an average coefficient of volume expansion of $3.0 \times 10^{-5} \text{ K}^{-1}$, by how much has the radius of Earth increased since the planet was formed?

The (average) temperature change of the Earth while forming was

$$\Delta T = 3000\text{ K} - 300\text{ K} = 2700\text{ K}$$

We are given the coefficient of volume expansion, but the problem is looking for a change in *length* so it will be easier to use the equation for linear expansion; so from Eq. 1.7 we get the *linear* coefficient

$$\alpha = \beta/3 = (3.0 \times 10^{-5} \text{ K}^{-1})/3 = 1.0 \times 10^{-5} \text{ K}^{-1}$$

Now use Eq. 1.5 with the radius of the Earth chosen as the length “ L ”. The *present* radius of the Earth is 6.37×10^6 m but since we expect the change in radius to be much smaller than this it is OK to use it for L . Then we get the change in radius:

$$\Delta L = L\alpha\Delta T = (6.37 \times 10^6\text{ m})(1.0 \times 10^{-5} \text{ K}^{-1})(2700\text{ K}) = 1.7 \times 10^5\text{ m} = 170\text{ km}$$

The change in radius of the Earth while it was forming was apparently 170 km (which is indeed small compared to 6370 km).

7. By how much does the volume of an aluminum cube 5.00 cm on an edge increase when the cube is heated from 10.0°C to 60.0°C ? [Use $\alpha_{\text{Al}} = 23 \times 10^{-6}/\text{C}^\circ$.]

From the given value of α (the *linear* expansion coefficient) and Eq. 1.7 we can get β , the *volume* expansion coefficient:

$$\beta = 3\alpha = 3(23 \times 10^{-6}/\text{C}^\circ) = 69 \times 10^{-6}/\text{C}^\circ .$$

Now, the volume of the cube is $V = (5.00\text{ cm})^3 = 125\text{ cm}^3$; this is really the initial volume, and as usual we don’t expect it to change much. The change in temperature of the cube is $\Delta T = 50.0^\circ\text{C}$. Eq. 1.6 tells us the corresponding increase in volume:

$$\Delta V = V\beta\Delta T = (125\text{ cm}^3)(69 \times 10^{-6}/\text{C}^\circ)(50.0^\circ\text{C}) = 4.3 \times 10^{-1}\text{ cm}^3 = 0.43\text{ cm}^3$$

The volume of the cube increases by 0.43 cm^3 .

1.2.3 Temperature and the Absorption of Heat

8. A certain substance has a mass per mole of 50 g/mole. When 314 J of heat is added to a 30.0 g sample of this material, its temperature rises from 25.0°C to 45.0°C. (a) What is the specific heat of this substance? (b) How many moles of the substance are present? (c) What is the molar specific heat of the substance?

(a) From the relation between heat transferred and temperature change, $Q = cm\Delta T$, we solve for c and use the given data. (The mass of the sample is $m = 0.0300$ kg and the change in temperature is $+20.0$ K.)

We find:

$$c = \frac{Q}{m\Delta T} = \frac{(314 \text{ J})}{(0.0300 \text{ kg})(20.0 \text{ K})} = 532 \frac{\text{J}}{\text{kg}\cdot\text{K}}$$

The specific heat of the substance is $532 \frac{\text{J}}{\text{kg}\cdot\text{K}}$

(b) We are given molar mass of the substance, which is the correspondence between mass and moles:

$$50 \text{ g} \longleftrightarrow 1 \text{ mol}$$

which we can use as a conversion factor to get:

$$30.0 \text{ g} = (30.0 \text{ g}) \left(\frac{(1.000 \text{ mol})}{(50.0 \text{ g})} \right) = 0.600 \text{ mol}$$

So 0.600 mol of the sample is present.

(c) To get c as a *molar* specific heat, redo part (a) using moles instead of mass:

$$c = \frac{Q}{n\Delta T} = \frac{(314 \text{ J})}{(0.600 \text{ mol})(20.0 \text{ K})} = 26.6 \frac{\text{J}}{\text{mol}\cdot\text{K}}$$

The molar specific heat of the substance is $26.6 \frac{\text{cal}}{\text{mol}\cdot\text{K}}$

1.2.4 Heats of Transformation